CHAPTER 2

Inequalities

Section 2A
Simple Inequalities

- 2-1 Graphing and Writing Inequalities
- 2-2 Solving Inequalities by Adding or Subtracting
- 2-3 Solving Inequalities by Multiplying or Dividing

Section 2B
Multi-Step and Compound Inequalities

- 2-4 Solving Two-Step and Multi-Step Inequalities
- 2-5 Solving Inequalities with Variables on Both Sides
- Algebra Lab Truth Tables and Compound Statements
- 2-6 Solving Compound Inequalities
- Connecting Algebra to Geometry Triangle Inequality
- 2-7 Solving Absolute-Value Inequalities

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 2

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
2-1 Lesson	2-2 Lesson	2-3 Lesson	Multi-Step Test Prep Ready to Go On?	2-4 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
2-5 Lesson	Hands-On Lab 2-6 Lesson	2-6 Lesson	2-6 Lesson Connecting Algebra to Geometry	2-7 Lesson
DAY 11	**DAY 12**	**Day 13**	**Day 13**	
2-7 Lesson	Multi-Step Test Prep Ready to Go On?	Chapter Review	Chapter Test	

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 2

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
2-1 Lesson 2-2 Lesson	2-3 Lesson Multi-Step Test Prep Ready to Go On?	2-4 Lesson 2-5 Lesson	Hands-On Lab 2-6 Lesson	2-6 Lesson Connecting Algebra to Geometry 2-7 Lesson
DAY 6	**DAY 7**			
2-7 Lesson Multi-Step Test Prep Ready to Go On?	Chapter Review Chapter Test			

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 2

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

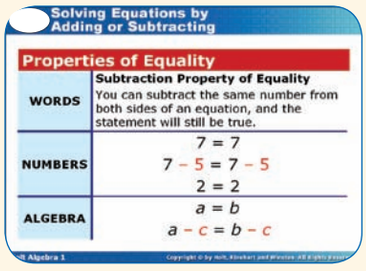
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

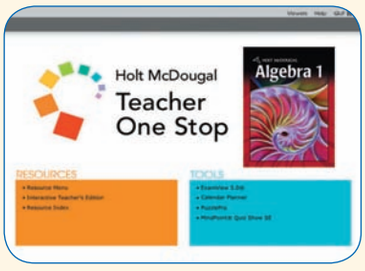
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 2

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Critical Thinking ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

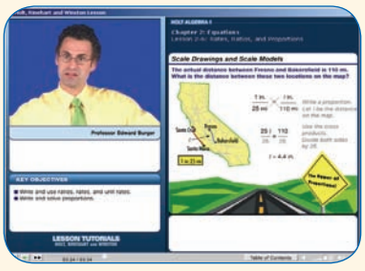
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

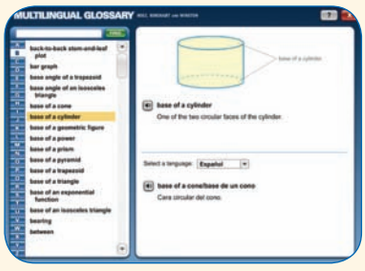
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



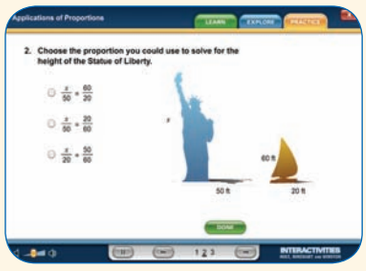
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 2

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!® Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!® Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView™ Assessment Suite

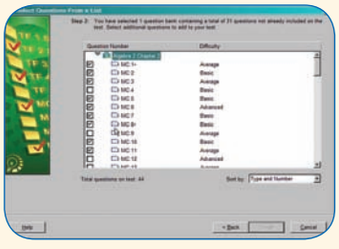
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
ExamView™ Assessment Suite
IDEA Works!® Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 2

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

MULTIPLE CHOICE

B Chapter Test

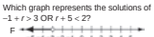
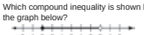
Select the best answer.

1. Describe the solutions of $4 \le n + 2$ in words.
   A all real numbers greater than 4
   B all real numbers greater than or equal to 2
   C all real numbers less than 2
   D all real numbers less than or equal to 4
2. Which graph represents $k < -1$?
   F
   G
   H
   J
3. Which graph represents $b \ge 5$?
   A
   B
   C
   D
4. Which inequality is shown by the graph below?

   F $x > -3$ H $x < -3$
   G $x \ge -3$ J $x \le -3$
5. Which inequality represents the situation "no more than 160 students are in the freshman class"?
   A $s > 160$ C $s < 160$
   B $s \ge 160$ D $s \le 160$
6. Solve $x + 7 > 2$.
   F $x > -5$ H $x > 5$
   G $x < -5$ J $x < 5$
7. Solve $m - 1.5 \le 4$.
   A $m \le -2.5$ C $m \le 5.5$
   B $m \ge -2.5$ D $m \ge 5.5$
8. Which inequality has the solutions shown below?

   F $p \ge 7$ H $5 > p - 2$
   G $5 < p - 2$ J $p \le 7$
9. Mike is on a cross-country trip and wants to drive at least 450 miles per day. So far today, he has driven 175 miles. Which inequality can be solved to show the number of miles m that Mike must drive to meet his daily goal?
   A $175 + m < 450$ C $175 + m > 450$
   B $175 + m \le 450$ D $175 + m \ge 450$
10. Solve $-28 < 4y$.
    F $y < -7$ H $y < 7$
    G $y > -7$ J $y > 7$
11. Solve $\frac{d}{2} \ge 8$.
    A $d \ge 4$ C $d \ge 16$
    B $d \le 4$ D $d \le 16$
12. Cookies are sold in the lunchroom for $1.50. Ana wants to buy cookies for a group of her friends. If she has $20, which inequality can be solved to show the number of cookies c she can buy?
    F $1.50c < 20$ H $1.50c > 20$
    G $1.50c \le 20$ J $1.50c \ge 20$
13. Solve $-4(x - 1) < 12$.
    A $x > -2$ C $x > -3$
    B $x < -2$ D $x < -3$

B Chapter Test *(continued)*

14. Solve $2^3 - a > -3(2 - 6)$.
    F $a < -6$ H $a < -4$
    G $a > -6$ J $a > -4$
15. John is considering accepting one of two sales positions. ABC Company offers a yearly salary of $45,000. XYZ Company offers a yearly salary of $38,000 plus a 2% annual commission on sales. For what amount of sales s is the salary at XYZ Company greater than the salary at ABC Company?
    A $s > 7000$ C $s > 70{,}000$
    B $s > 35{,}000$ D $s > 350{,}000$
16. Solve $3(1 + h) \le 3h + 9$.
    F $h \le 1$ H no solutions
    G $h \le 2$ J all real numbers
17. Solve $\frac{3}{4}s \ge \frac{1}{4}s + 8$.
    A $s \ge 4$ C $s \ge 12$
    B $s \ge 8$ D $s \ge 16$
18. Jasmine and her sister are saving to buy MP3 players. Jasmine has $50 and plans to save $10 per week. Her sister has $80 and plans to save $7 per week. In how many weeks will Jasmine have more money saved than her sister?
    F 2 weeks H 10 weeks
    G 4 weeks J 11 weeks
19. Solve the compound inequality $-2 \le m + 3 < 13$.
    A $-5 \le m < 10$
    B $-2 \le m < 13$
    C $1 \le m < 16$
    D $6 \le m < 39$
20. Which graph represents the solutions of $-1 + r > 3$ OR $r + 5 < 2$?
    F 
    G
    H
    J
21. Which compound inequality is shown by the graph below?
    
    A $x \ge -3$ AND $x > 3$
    B $x \ge -3$ AND $x < 3$
    C $x \ge -3$ OR $x > 3$
    D $x \ge -3$ OR $x < 3$
22. Which represents the solution of $|2x| - 5 < -1$?
    F $x > -2$ OR $x < 2$
    G $x < -2$ OR $x > 2$
    H $x > -2$ AND $x < 2$
    J $x > 2$ AND $x < -2$
23. Which represents the solution of $3|x| + 3 > 12$?
    A $x > 3$ OR $x < -3$
    B $x < 3$ OR $x > -3$
    C $x > -3$ AND $x > 3$
    D $x > -3$ AND $x < -3$

A Chapter Test

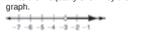
C Chapter Test

FREE RESPONSE

B Chapter Test

1. Describe the solutions of $\frac{1}{3}x \ge 2$ in words.

Graph each inequality.

2. $k \le -1$
3. $w > 2\frac{1}{2}$
4. Write the inequality shown by the graph.
   
5. Citizens less than 18 years old are not allowed to vote. Define a variable and write an inequality for the ages of citizens who are not allowed to vote.

Solve each inequality and graph the solutions.

6. $5 < x + 31$
7. $a - \frac{1}{4} > 5\frac{3}{4}$
8. $-5 + b \le -19$
9. A store manager is accepting applications for part-time workers. He can hire no more than 14 people. So far, he has hired 9 people. Write and solve an inequality to determine how many more people the manager can hire.

Solve each inequality.

10. $-4x < -64$
11. $\frac{4}{5}h \ge -2$

B Chapter Test *(continued)*

12. A designer is creating shirts that each have 12 buttons. She bought a container of 115 buttons and plenty of fabric. What are the possible numbers of shirts she can make?

Solve each inequality.

13. $\frac{x - 3}{-7} \le 8$
14. $-12 + 3x - 3^2 < 18$
15. One salesperson earns $1600 per month. A second salesperson earns $500 plus 5% of their sales per month. For what amount of sales would the second salesperson earn more than the first?

Solve each inequality.

16. $-3(x - 1) > -3x - 2$
17. $\frac{4}{5}x + \frac{1}{3} \le -1$
18. Perfect Pipes charges $120 plus $40 an hour to make plumbing repairs. No Leaks Here charges $50 plus $60 an hour. For how many hours is Perfect Pipes less expensive than No Leaks Here?

Solve each compound inequality and graph the solutions.

19. $-8 \le c + 5 < -1$
20. $-a + 8 < -2$ OR $-3a > -9$
21. Write the compound inequality shown by the graph.

Solve each absolute-value inequality.

22. $|3x| - 10 < -4$
23. $2|x| + 3 > 18$

PERFORMANCE ASSESSMENT

Chapter Test

Grocery Store Game

Purpose

This performance task assesses the student's ability to write and solve compound inequalities, specifically intersections.

Time

30–45 minutes

Grouping

Individuals or partners

Preparation Hints

Review the two types of compound inequalities, intersections and unions, and remind students of some of the application problems where intersections were used to find an unknown value in a given range.

Overview

This task uses a game from a popular TV show in which contestants try to buy between $20 and $21 of grocery-store items. Compound inequalities are used to figure out prices and quantities that would meet the target criteria. Students also use number sense to round appropriately and find values within the range of the inequality that meet given conditions.

Introduce the Task

Ask students if they have ever seen the Grocery Game on *The Price is Right*. If students are familiar with the games on *The Price is Right*, ask what other games could be described by inequalities. Examples include Cliff Hangers (the sum of differences between guessed prices and actual prices must be less than $25), the Range Game (the actual price must be in a range of $600), and Shopping Spree (the sum of three out of four prizes must be greater than a target price).

Performance Indicators

_____ Writes compound inequalities.
_____ Solves compound inequalities.
_____ Explains how to round values given the context of the problem.
_____ Explains how to graph points and/or inequalities on a number line.
_____ Identifies whole-number or decimal values that satisfy conditions.

Scoring Rubric

Level 4: Student solves problems correctly and gives good explanations.
Level 3: Student solves problems but does not give satisfactory explanations.
Level 2: Student solves some problems but does not give satisfactory explanations.

Chapter Test *(continued)*

Grocery Store Game

Justin is a lucky guy! He has a chance to win a trip on the TV game show *You Can Price Like a Pro*. All he has to do is win the Grocery Store Game. In this game, Justin is shown six grocery-store items without being shown the prices. He must buy multiples of items until he has a total between $20 and $21 inclusive. If he doesn't reach $20 with his first item, he continues buying items and adding to his total, until he either wins or goes over $21.

1. Justin begins by buying six bottles of hand lotion.
   a. Write a compound inequality to find the range of prices for one bottle of lotion that would make Justin win instantly.
   b. Solve the compound inequality. Round each answer to the hundredths place.
   c. In your work for part b, you should have gotten a repeating decimal. Explain how you decided whether to round this number up or down.
   d. Explain how you would graph the solutions in part b on a number line.
2. One bottle of lotion actually costs $2.99. That gives Justin a running total of $17.94. He now considers buying packages of gum.
   a. Justin guesses that one package of gum costs $0.50. Write a compound inequality to find how many packages he should buy.
   b. Solve your compound inequality from part a. Write your answer as a compound inequality.
   c. Justin can only buy whole packages of gum. Assuming each package actually costs $0.50, how many could he buy to win the game?
3. Justin plays it safe and buys four packages of gum, and he wins!
   a. Write a compound inequality to find the range of actual prices for one package of gum.
   b. Solve your compound inequality from part a. Don't round your answers.
   c. If the actual price of one package of gum ends with a 9, what could the price have been?

CHAPTER 2

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

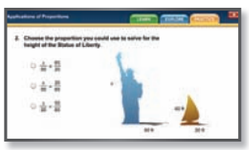
SECTION 2A
Simple Inequalities

MULTI-STEP TEST PREP Students write, solve, and graph inequalities to model the cost of travel, lodging, and entertainment while on a vacation.

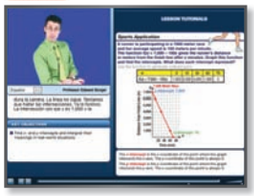
SECTION 2B
Multi-Step and Compound Inequalities

MULTI-STEP TEST PREP Students write, solve, and graph inequalities to determine the costs associated with recording a CD at a studio.

Interactivities Online ▶



Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 2

Inequalities

Sam Dudgeon/HMH

COMMON CORE

2A Simple Inequalities

2-1 Graphing and Writing Inequalities — A.REI.3
2-2 Solving Inequalities by Adding or Subtracting — A.REI.3
2-3 Solving Inequalities by Multiplying or Dividing — A.REI.3

2B Multi-Step and Compound Inequalities

2-4 Solving Two-Step and Multi-Step Inequalities — A.REI.3
2-5 Solving Inequalities with Variables on Both Sides — A.REI.3
Lab Truth Tables and Compound Statements — A.REI.3
2-6 Solving Compound Inequalities — A.REI.3
2-7 Solving Absolute-Value Inequalities — A.REI.3

Chapter Focus

- Solve multi-step inequalities.
- Write and solve inequalities to solve problems.

For a *Good* Cause

You can use the concepts in this chapter to plan for a fund-raising event. Inequalities help you determine how to reach your fund-raising goals.

Learn It Online
Chapter Project Online

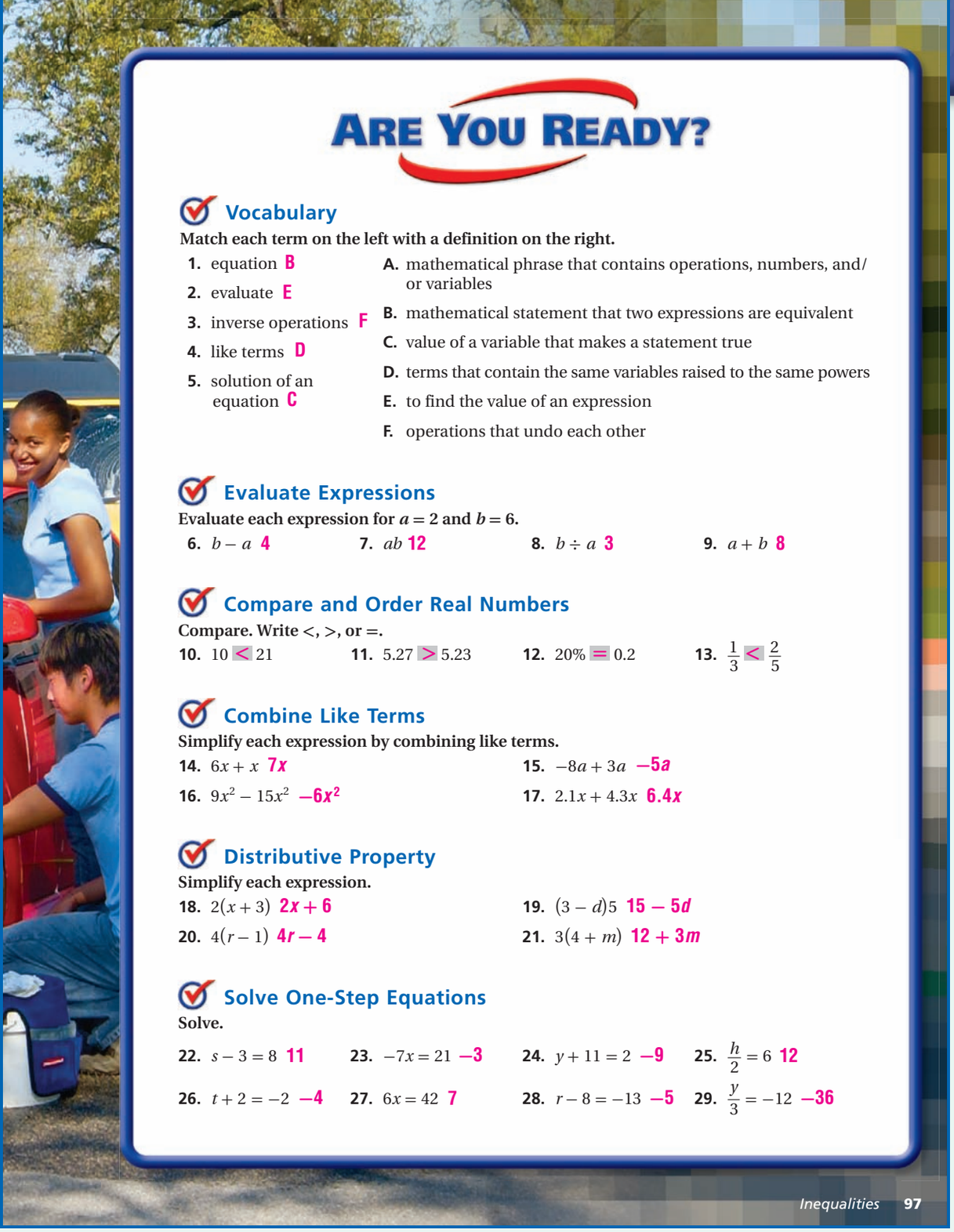
Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. equation **B**
2. evaluate **E**
3. inverse operations **F**
4. like terms **D**
5. solution of an equation **C**

A. mathematical phrase that contains operations, numbers, and/or variables

B. mathematical statement that two expressions are equivalent

C. value of a variable that makes a statement true

D. terms that contain the same variables raised to the same powers

E. to find the value of an expression

F. operations that undo each other

Evaluate Expressions

Evaluate each expression for $a = 2$ and $b = 6$.

6. $b - a$ **4**
7. ab **12**
8. $b \div a$ **3**
9. $a + b$ **8**

Compare and Order Real Numbers

Compare. Write <, >, or =.

10. $10 < 21$
11. $5.27 > 5.23$
12. $20\% = 0.2$
13. $\frac{1}{3} < \frac{2}{5}$

Combine Like Terms

Simplify each expression by combining like terms.

14. $6x + x$ **$7x$**
15. $-8a + 3a$ **$-5a$**
16. $9x^2 - 15x^2$ **$-6x^2$**
17. $2.1x + 4.3x$ **$6.4x$**

Distributive Property

Simplify each expression.

18. $2(x + 3)$ **$2x + 6$**
19. $(3 - d)5$ **$15 - 5d$**
20. $4(r - 1)$ **$4r - 4$**
21. $3(4 + m)$ **$12 + 3m$**

Solve One-Step Equations

Solve.

22. $s - 3 = 8$ **11**
23. $-7x = 21$ **-3**
24. $y + 11 = 2$ **-9**
25. $\frac{h}{2} = 6$ **12**
26. $t + 2 = -2$ **-4**
27. $6x = 42$ **7**
28. $r - 8 = -13$ **-5**
29. $\frac{y}{3} = -12$ **-36**

CHAPTER **2**

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

***Are You Ready? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Prerequisite Skill	*Are You Ready? Intervention,* Chapter 2 Worksheets	*Are You Ready? Intervention,* Chapter 2 Online	
Evaluate Expressions	Skill 60	Activity 60	Diagnose and Prescribe Online
Compare and Order Real Numbers	Skill 16	Activity 16	
Combine Like Terms	Skill 57	Activity 57	
Distributive Property	Skill 56	Activity 56	
Solve One-Step Equations	Skill 68	Activity 68	

***Are You Ready? Enrichment,* Chapter 2**

Worksheets

Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. An inequality states that two things are not equal.
2. A compound inequality is made of two or more simple inequalities.
3. The intersection of two graphs is the place where the two graphs overlap.
4. *Unite, unify, unit, unicycle, uniform, universal;* each word has the word *one* in its definition.

CHAPTER 2 Study Guide: Preview

Where You've Been

Previously, you

- learned the properties of equality.
- solved equations by using inverse operations.
- solved equations with variables on both sides.

In This Chapter

You will study

- the properties of inequality.
- how to solve inequalities by using inverse operations.
- how to solve inequalities with variables on both sides.
- how to solve compound inequalities.

Where You're Going

You can use the skills in this chapter

- in all your future math classes, including Geometry.
- in other classes, such as Health, Chemistry, Physics, and Economics.
- in the real world to plan a budget, to find cost-efficient services, and to set financial goals.

Key Vocabulary/Vocabulario

compound inequality	desigualdad compuesta
inequality	desigualdad
intersection	intersección
solution of an inequality	solución de una desigualdad
union	unión

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The prefix *in-* means "not." An *equality* states that two things are equal. Use these meanings to write your own definition for the word **inequality**.
2. The word *compound* means "consisting of two or more parts." What do you think a **compound inequality** might be?
3. The **intersection** of two roads is the place where the two roads overlap. What do you think the *intersection* of two graphs would be?
4. The word **union** begins with the root *uni-*. List some other words that begin with *uni-*. What do all of these words have in common?

MATHEMATICAL PRACTICES

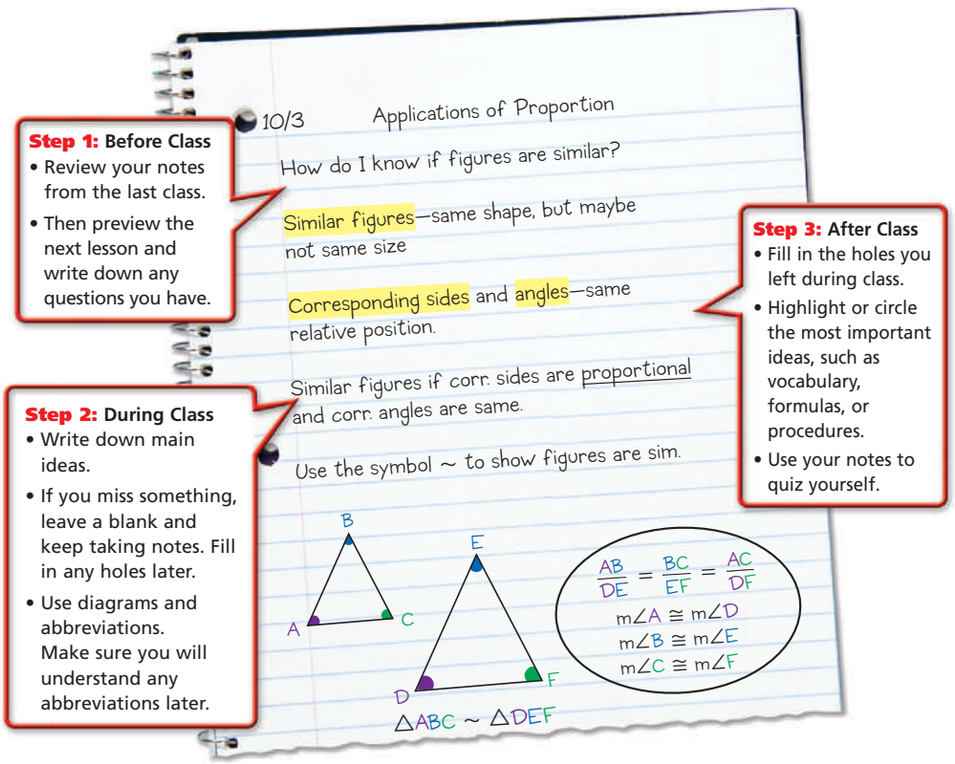
The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 104–105, 107, 109–110, 115–116, 118, 123–124, 129–131, 138–139, 145–146, 148
2. **Reason abstractly and quantitatively.** Pages 110, 117, 132, 139–140, 146
3. **Construct viable arguments and critique the reasoning of others.** Pages 102, 104–105, 109, 110–111, 114, 115–117, 122, 125, 129, 131–132, 137, 139–140, 145, 147
4. **Model with mathematics.** Pages 102, 103–104, 107, 108, 109, 114, 115, 122, 127, 134, 144
5. **Use appropriate tools strategically.** Pages 107, 113, 133
6. **Attend to precision.** Pages 110, 117
7. **Look for and make use of structure.** Pages 101, 106, 111, 112, 113, 142, 143
8. **Look for and express regularity in repeated reasoning.** Page 133

98 Chapter 2

Study Strategy: Use Your Notes Effectively

Taking notes helps you arrange, organize, and process information from your textbook and class lectures. In addition to taking notes, you need to use your notes before and after class effectively.



Try This

1. Look at the next lesson in your textbook. Write down some questions you have about the material in that lesson. Leave space between each question so that you can write the answers during the next class.
2. Look at the notes you took during the last class. List three ways you can improve your note-taking skills.

Standards for Mathematical Content High School

CREATING EQUATIONS Create equations that describe numbers or relationships		LESSON
CC.9-12.A.CED.1	Create equations and inequalities in one variable and use them to solve problems.*	2-3, 2-5, 2-7
REASONING WITH EQUATIONS AND INEQUALTIIES Solve equations and inequalities in one variable		
CC.9-12.A.REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	2-1, 2-2, 2-3, 2-4, 2-5, Lab 2-6, 2-6, 2-7

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

Online Edition

Study Strategy: Use Your Notes Effectively

Discuss Notes are most helpful when they are well organized. Students may wish to divide their notebooks into sections for class notes, definitions, formulas, homework, scrap work, review, etc.

Extend Abbreviations can be used when taking notes to save space and time. Some students use *b/c* for *because* or *s/t* for *sometimes*. Ask students to share other abbreviations they use.

Answers to *Try This*

1–2. Check students' work.

Reading Connection

Mathematical Mysteries
by Calvin C. Clawson

Humans began counting at least 35,000 years ago, according to this fascinating inquiry into the wonder of numbers. Counting and ordering numbers allowed early humans to establish that one set of objects was larger than another, and the concept of inequality was born. The author's account of the first million years or so of numbers, plus chapters on sequences and series, prime numbers, and a host of other algebra-related topics make for entertaining and enlightening reading.

Activity Ask students to read, Chapter 1 "Discovery of the Number Sequence," and then to report on the significance of a baboon thigh bone with 29 notches found in Africa's Lebembo Mountains.

SECTION
2A Simple Inequalities

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 2-1 Graphing and Writing Inequalities • Identify solutions of inequalities with one variable. • Write and graph inequalities with one variable. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Lesson 2-2 Solving Inequalities by Adding or Subtracting • Solve one-step inequalities by using addition. • Solve one-step inequalities by using subtraction. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** index cards
Lesson 2-3 Solving Inequalities by Multiplying or Dividing • Solve one-step inequalities by using multiplication. • Solve one-step inequalities by using division. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Technology Lab	

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 11, 13, 15, 17, 19, 21, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 51, 53, 59, 61, 65

Lesson 2: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 35a, 35b, 35c, 37, 39, 41, 43, 45

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 57, 59, 67, 71, 73, 75

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

INEQUALITIES

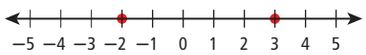
Lessons 2-1 to 2-2

One of the basic building blocks of mathematics is the Law of Trichotomy. This axiom says that given any two real numbers, a and b, exactly one of the following is true:

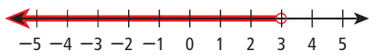
$$a = b,\ a < b,\ \text{or}\ a > b.$$

In the first case, a and b are related by an equality. In the second and third cases, a and b are related by an *inequality.*

To understand inequalities, it is necessary to understand their connection to the real number line. For example, the inequality $-2 < 3$, which states that -2 is less than 3, means that -2 is to the left of 3 on the number line.



A *solution* of an algebraic inequality is any value of the variable that makes the inequality true. Thus, the solution set of $x < 3$ contains every value on the number line to the left of 3, since any of these values, when substituted in the inequality, result in a true statement. This solution set, consisting of infinitely many elements, can be shown as a ray on the number line. (The empty circle at 3 indicates that $x = 3$ is not a solution.)



SOLVING INEQUALITIES

Lessons 2-3 to 2-4

Solving an inequality in one variable is similar to solving an equation in one variable. The goal is to isolate the variable on one side of the inequality by writing a series of equivalent inequalities—that is, a series of inequalities with the same solution set.

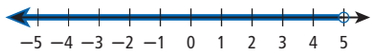
The Addition and Subtraction Properties of Inequality are analogous to their counterparts for equations: the same quantity may be added to or subtracted from both sides of an inequality. For multiplication and division there is an important caveat.

Consider the inequality $a > b$. By the Subtraction Property of Inequality, $a - a > b - a$ or, equivalently, $0 > b - a$. Applying the Subtraction Property of Inequality again shows that $0 - b > b - a - b$ or $-b > -a$, which is the same as $-a < -b$. In other words, when both sides of the inequality are multiplied by -1, the direction of the inequality is reversed. More generally, the direction of the inequality is reversed when both sides of an inequality are multiplied or divided by any negative number.

TRANSFORMATIONS

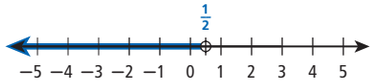
Lessons 2-5 to 2-6

We can use transformations to examine solutions of inequalities. For example, compare the graph of $x < 3$ with the graph of $x - 2 < 3$.



Notice that the graphs are identical except that the graph of $x - 2 < 3$ has been shifted 2 units to the right. In general, the graph of $x - a < k$ is the same as that of $x < k$, but shifted a units to the right (if $a > 0$) or $|a|$ units to the left (if $a < 0$).

The coefficient of x represents the stretching or shrinking of the graph. For example, the graph of $6x < 3$ is shown below.



As compared to the graph of $x < 3$, the ray is contracted toward the origin by a factor of 6.

Finally, the graph of $-x < 3$ is shown below.

Comparing this to the graph of $x < 3$ shows that the negative sign in front of x reflects the ray about the origin.

2-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Identify solutions of inequalities in one variable.

Write and graph inequalities in one variable.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Compare. Write <, >, or =.

1. -3 < 2 **2.** 6.5 > 6.3

3. $\frac{1}{2}$ > $-\frac{3}{4}$ **4.** 0.25 = $\frac{1}{4}$

Tell whether the inequality $x < 5$ is true or false for the following values of x.

5. $x = -10$ T **6.** $x = 5$ F

7. $x = 4.99$ T **8.** $x = -\frac{1}{5}$ T

Also available online

Math Humor

Q: Why did the Moore family name their son Lester?

A: So he could be called either "Moore" or "Les."

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online
State Resources Online

COMMON CORE

2-1 Graphing and Writing Inequalities

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Objectives
Identify solutions of inequalities in one variable.

Write and graph inequalities in one variable.

Vocabulary
inequality
solution of an inequality

Who uses this?

Members of a crew team can use inequalities to be sure they fall within a range of weights. (See Example 4.)

The athletes on a lightweight crew team must weigh 165 pounds or less. The acceptable weights for these athletes can be described using an *inequality*.

An **inequality** is a statement that two quantities are not equal. The quantities are compared by using one of the following signs:

$<$	$>$	$\leq$	$\geq$	$\neq$
$A < B$	$A > B$	$A \leq B$	$A \geq B$	$A \neq B$
A is less than B.	A is greater than B.	A is less than or equal to B.	A is greater than or equal to B.	A is not equal to B.

A **solution of an inequality** is any value of the variable that makes the inequality true.

EXAMPLE 1 Identifying Solutions of Inequalities

Describe the solutions of $3 + x < 9$ in words.

Test values of x that are positive, negative, and 0.

x	−2.75	0	5.99	6	6.01	6.1
$3 + x$	0.25	3	8.99	9	9.01	9.1
$3 + x \stackrel{?}{<} 9$	$0.25 \stackrel{?}{<} 9$	$3 \stackrel{?}{<} 9$	$8.99 \stackrel{?}{<} 9$	$9 \stackrel{?}{<} 9$	$9.01 \stackrel{?}{<} 9$	$9.1 \stackrel{?}{<} 9$
Solution?	Yes	Yes	Yes	No	No	No

When the value of x is a number less than 6, the value of 3 + x is less than 9.
When the value of x is 6, the value of 3 + x is equal to 9.
When the value of x is a number greater than 6, the value of 3 + x is greater than 9.

The solutions of $3 + x < 9$ are numbers less than 6.

Writing Math

The solutions of the inequality in Example 1 can be written in *set-builder notation* as $\{x \mid x < 6\}$, which is read as "the set of all real numbers x such that x is less than 6."

CHECK IT OUT! **1.** Describe the solutions of $2p > 8$ in words.

all real numbers greater than 4

© Charles Crust

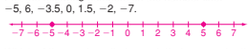
1 Introduce

EXPLORATION

Graphing and Writing Inequalities

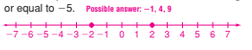
You can use a number line to help you see relationships among numbers.

1. Plot the following numbers on the number line: −5, 6, −3.5, 0, 1.5, −2, −7.



2. Which of the numbers that you plotted are greater than or equal to −5? −5, 6, −3.5, 0, 1.5, −2

3. Name and plot three additional numbers that are greater than or equal to −5. Possible answer: −1, 4, 9



4. Draw a ray connecting all the numbers that are greater than or equal to −5.

THINK AND DISCUSS

5. **Describe** how the ray would be different if you plot and connect numbers that are less than or equal to −5. The ray would point in the opposite direction.

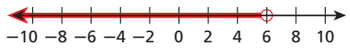
6. **Explain** how you could draw a ray to show all numbers that are greater than or equal to $4\frac{1}{2}$. Plot $4\frac{1}{2}$, and then draw a ray that points to the right and has its endpoint at $4\frac{1}{2}$.

Motivate

Make some statements like "There are fewer than 30 students in this class" and "The temperature will be at least 70 degrees tomorrow." Ask students what the phrases "fewer than" and "at least" mean. Ask students for other phrases that indicate a comparison.

Explorations and answers are provided in your online resources.

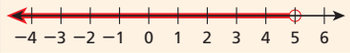
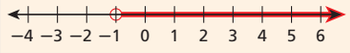
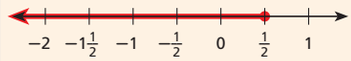
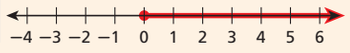
An inequality like $3 + x < 9$ has too many solutions to list. You can use a graph on a number line to show all the solutions.



The solutions are shaded and an arrow shows that the solutions continue past those shown on the graph. To show that an endpoint is a solution, draw a solid circle at the number. To show that an endpoint is not a solution, draw an empty circle.

Know it! Note

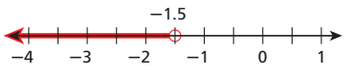
Graphing Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers less than 5	$x < 5$	
All real numbers greater than −1	$x > -1$	
All real numbers less than or equal to $\frac{1}{2}$	$x \leq \frac{1}{2}$	
All real numbers greater than or equal to 0	$x \geq 0$	

EXAMPLE 2 **Graphing Inequalities**

Graph each inequality.

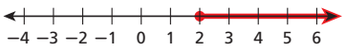
A $b < -1.5$



Draw an empty circle at −1.5.

Shade all the numbers less than −1.5 and draw an arrow pointing to the left.

B $r \geq 2$



Draw a solid circle at 2.

Shade all the numbers greater than 2 and draw an arrow pointing to the right.

CHECK IT OUT!

Graph each inequality.

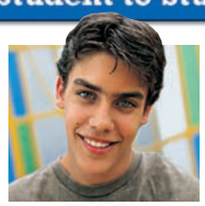
2a. $c > 2.5$ **2b.** $2^2 - 4 \geq w$ **2c.** $m \leq -3$

2a.

2b.

2c.

Student to Student — *Graphing Inequalities*

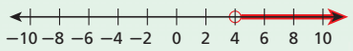


Victor Solomos
Palmer High School

To know which direction to shade a graph, I write inequalities with the variable on the left side of the inequality symbol. I know that the symbol has to point to the same number after I rewrite the inequality.

For example, I write $4 < y$ as $y > 4$.

Now the inequality symbol points in the direction that I should draw the shaded arrow on my graph.



Digital Vision/getty/images

2 Teach

Guided Instruction

Review the inequality symbols before identifying solutions of inequalities. Focus on building an understanding of inequalities, rather than solving inequalities. Procedures for solving will be addressed in subsequent lessons.

Teaching Tip **Reading Math** Before **Example 4,** have students use the phrases "no more than," "no less than," "at most," and "at least" in everyday sentences.

ENGLISH LANGUAGE LEARNERS

Reaching All Learners *Through Cognitive Strategies*

Use the DISC method to graph inequalities:

***D**raw*—Draw a number line and a circle at the endpoint.

***I**nclude?*—Look at the inequality sign to determine whether the circle should be solid or empty.

***S**hade*—Shade in the correct direction.

***C**heck*—Substitute a value on the solution side into the inequality to check that the inequality is true.

COMMON ERROR ALERT

When students find that $x = 6$ does not make the inequality in **Example 1** true, some students believe the solution should be numbers less than 5. Remind them of the numbers between 5 and 6.

Power Presentations with PowerPoint®

Additional Examples

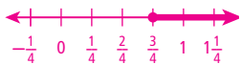
Example 1

Describe the solutions of $x - 6 \geq 4$ in words. all real numbers greater than or equal to 10

Example 2

Graph each inequality.

A. $m \geq \frac{3}{4}$



B. $t < 5(-1 + 3)$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- Why is it important to test values for the variable that are positive, negative, and zero?

EXAMPLE 2

- How do you decide which way to shade the graph?

Teaching Tip **Communicating Math** Some people refer to a solid circle as "closed" and an empty circle as "open."

Teaching Tip **Geometry** A *ray* is a part of a line that extends indefinitely in one direction from an endpoint. Many of the graphs in these exercises are rays.

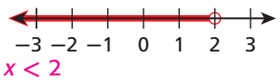
Power Presentations with PowerPoint®

Additional Examples

Example 3

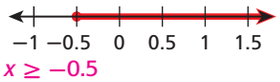
Write the inequality shown by each graph.

A.



$x < 2$

B.



$x \geq -0.5$

Example 4

Ray's dad told him not to turn on the air conditioner unless the temperature is at least 85 °F. Define a variable and write an inequality for the temperatures at which Ray can turn on the air conditioner. Graph the solutions.

t = temperature in °F; $t \geq 85$

Also available online

Questioning Strategies

EXAMPLE 3

- What does an empty or solid circle tell you? What does the direction of the arrow tell you?

EXAMPLE 4

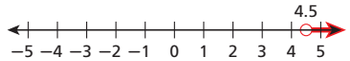
- Do you think all the values that were shaded are reasonable answers in this situation?

Teaching Tip **Critical Thinking** Discuss with students what types of problems might have answers that can only be whole numbers, positive numbers, etc.

EXAMPLE 3 Writing an Inequality from a Graph

Write the inequality shown by each graph.

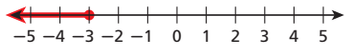
A



Use any variable. The arrow points to the right, so use either > or ≥.
The empty circle at 4.5 means that 4.5 is not a solution, so use >.

$h > 4.5$

B



Use any variable. The arrow points to the left, so use either < or ≤.
The solid circle at −3 means that −3 is a solution, so use ≤.

$m \leq -3$

CHECK IT OUT! **3.** Write the inequality shown by the graph.

$x < 2.5$

EXAMPLE 4 *Sports Application*

The members of a lightweight crew team can weigh no more than 165 pounds each. Define a variable and write an inequality for the acceptable weights of the team members. Graph the solutions.

Reading Math

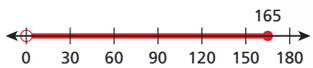
"No more than" means "less than or equal to."

"At least" means "greater than or equal to."

Let w represent the weights that are allowed.

Athletes may weigh	no more than	165 pounds.
w	$\leq$	165

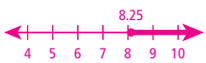
$w \leq 165$



Stop the graph at 0 because a person's weight must be a positive number.

CHECK IT OUT! **4.** A store's employees earn at least $8.25 per hour. Define a variable and write an inequality for the amount the employees may earn per hour. Graph the solutions.

d = amount employee can earn per hour; $d \geq 8.25$

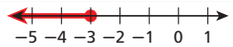


Know it! Note

THINK AND DISCUSS

1. Compare the solutions of $x > 2$ and $x \geq 2$.

2. GET ORGANIZED Copy and complete the graphic organizer. Draw a graph in the first row and write the correct inequality in the second row.

Inequality	Graph
$x > 1$	
	

3 Close

Summarize

Make sure students can describe the solutions of inequalities in words. Stress the need to test positive and negative values as well as zero. Remind students that there are many numbers between consecutive integers.

When graphing inequalities that represent real-world situations, students should consider which values are reasonable in the context.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Both graphs include the real numbers greater than 2. The graph of $x \geq 2$ also includes 2.
2. See Additional Answers.

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GUIDED PRACTICE

1. **Vocabulary** How is a *solution of an inequality* like a solution of an equation?
A solution of an inequality makes the inequality true when substituted for the variable.

SEE EXAMPLE 1

Describe the solutions of each inequality in words.

2. $g - 5 \geq 6$ 3. $-2 < h + 1$ 4. $20 > 5t$ 5. $5 - x \leq 2$

SEE EXAMPLE 2

Graph each inequality.

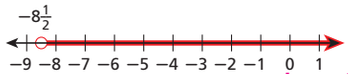
6. $x < -5$ 7. $c \geq 3\frac{1}{2}$ 8. $(4-2)^3 > m$ 9. $p \geq \sqrt{17+8}$

SEE EXAMPLE 3

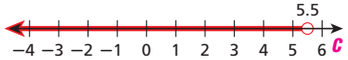
Write the inequality shown by each graph.

10.

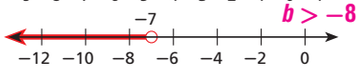
$a \leq -2$

11. 

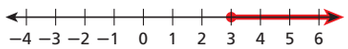
$b > -8\frac{1}{2}$

12. 

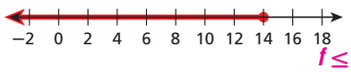
$c < 5.5$

13. 

$d < -7$

14. 

$e \geq 3$

15. 

$f \leq 14$

SEE EXAMPLE 4

Define a variable and write an inequality for each situation. Graph the solutions.

16. There must be at least 20 club members present in order to hold a meeting.
m = members present; $m \geq 20$ where m is a whole number

17. A trainer advises an athlete to keep his heart rate under 140 beats per minute.
r = heart rate; $r < 140$ where r is positive

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22–25	2
26–31	3
32–33	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

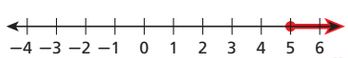
Describe the solutions of each inequality in words.

18. $-2t > -8$ 19. $0 > w - 2$ 20. $3k > 9$ 21. $\frac{1}{2}b \leq 6$

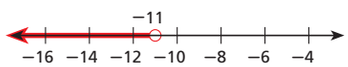
Graph each inequality.

22. $7 < x$ 23. $t \leq -\frac{1}{2}$ 24. $d > 4(5-8)$ 25. $t \leq 3^2 - 2^2$

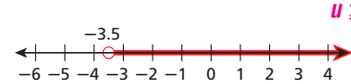
Write the inequality shown by each graph.

26. 

$u \geq 5$

27. 

$v < -11$

28. 

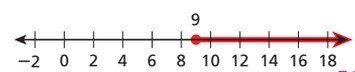
$w > -3.5$

29.

$x > -3.3$

30.

$y < 4$

31. 

$z \geq 9$

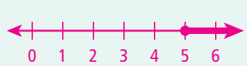
Define a variable and write an inequality for each situation. Graph the solutions.

32. The maximum speed allowed on Main Street is 25 miles per hour.
s = speed; $s \leq 25$ where s is nonnegative

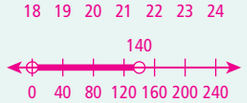
33. Applicants must have at least 5 years of experience.
y = years of experience; $y \geq 5$

7.

8.

9. 

16.

17. 

18. all real numbers less than 4

19. all real numbers less than 2

20. all real numbers greater than 3

21. all real numbers less than or equal to 12

22.

23.

24.

25.

32.

33.

2-1 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 18–25, 34–41, 50–53
Average 18–25, 34–41, 62–64, 67
Advanced 18–25, 34–41, 62–66

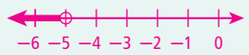
If you finished Examples **1–4**
Basic 18–45, 50–53, 56–61
Average 18–33, 34–48 even, 54–68
Advanced 18–33, 38–45, 54–56, 58–69

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 22, 28, 32, 40, 42

Teaching Tip

Inclusion For exercises like **Exercises 32–33,** have students ask themselves a question to decide whether the endpoint is a solution. For example, for **Exercise 32,** ask "Is 25 miles per hour allowed?" Yes, so the graph should have a solid circle.

Answers

2. all real numbers greater than or equal to 11
3. all real numbers greater than −3
4. all real numbers less than 4
5. all real numbers greater than or equal to 3
6. 

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 55, 56

Construct viable arguments and critique the reasoning of others. Exercises 54, 57–58, 69

Model with mathematics. Exercises 16–17, 32–33, 42–45

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MULTI-STEP TEST PREP **Exercise 55** involves using inequalities to describe how much money can be spent shopping. This exercise prepares students for the Multi-Step Test Prep.

Answers

34. x is greater than 7.
35. h is less than -5.
36. d is less than or equal to 23.
37. r is greater than or equal to -2.
38.

39.

40.

41.

42.

43.

44.

45.

46. x represents the distance in miles between two locations.
47. x represents the age in years of a child at a childcare center when x is positive.
48. x represents the hour on an analog clock when x is a natural number.
49. x represents the number of millions of albums sold by a popular band.

Write each inequality in words.

34. $x > 7$ **35.** $h < -5$ **36.** $d \leq 23$ **37.** $r \geq -2$

Write each inequality with the variable on the left. Graph the solutions.

38. $19 < g$ **$g > 19$** **39.** $17 \geq p$ **$p \leq 17$** **40.** $10 < e$ **$e > 10$** **41.** $0 < f$ **$f > 0$**

Define a variable and write an inequality for each situation. Graph the solutions.

42. The highest temperature ever recorded on Earth was 135.9 °F at Al Aziziyah, Libya, on September 13, 1922. **t = temperature; $t \leq 135.9$**

43. Businesses with profits less than $10,000 per year will be shut down. **p = profits; $p < 10{,}000$**

44. You must be at least 46 inches tall to ride a roller coaster at an amusement park. **h = height; $h \geq 46$**

45. Due to a medical condition, a hiker can hike only in areas with an elevation no more than 5000 feet above sea level. **e = elevation; $e \leq 5000$**

Write a real-world situation that could be described by each inequality.

46. $x \geq 0$ **47.** $x < 10$ **48.** $x \leq 12$ **49.** $x > 8.5$

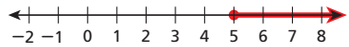
Match each inequality with its graph.

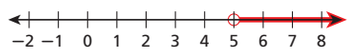
50. $x \geq 5$ **A**

51. $x < 5$ **D**

52. $x > 5$ **B**

53. $x \leq 5$ **C**

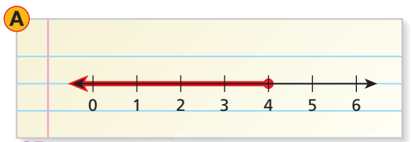
A. 

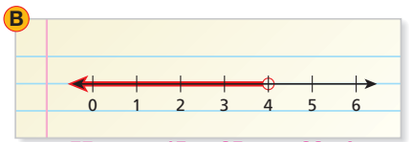
B. 

C.

D.

54. ERROR ANALYSIS Two students graphed the inequality $4 > b$. Which graph is incorrect? Explain the error. **A is incorrect; it should be drawn with an empty circle.**

A 

B 

55a. **$125 - s \geq 90$; $s \leq 35$ where s is nonnegative**

55b.

55c. **$s + 15 \leq 35$; $s \leq 20$ where s is nonnegative**

MULTI-STEP TEST PREP

55. **a.** Mirna earned $125 baby-sitting during the spring break. She needs to save $90 for the German Club trip. She wants to spend the remainder of the money shopping. Write an inequality to show how much she can spend.

b. Graph the inequality you wrote in part **a.**

c. Mirna spends $15 on a bracelet. Write an inequality to show how much money she has left to spend.

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PRACTICE A

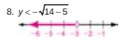
PRACTICE C

PRACTICE B

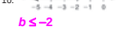
Describe the solutions of each inequality in words.

1. $2m \geq 6$ all real numbers greater than or equal to 3
2. $t + 3 < 8$ all real numbers less than 5
3. $1 < x - 5$ all real numbers greater than 6
4. $-10 \geq \frac{1}{2}c$ all real numbers less than or equal to –20

Graph each inequality.

5. $x > -7$
6. $p \geq 2^3$
7. $4.5 \geq r$
8. $y < -\sqrt{14-5}$ 

Write the inequality shown by each graph.

9.
$a > 6$
10. 
$b \leq -2$
11.
$c < 8.5$
12.
$d \geq 45$

Define a variable and write an inequality for each situation. Graph the solutions.

13. Josephine sleeps more than 7 hours each night.
s = hours of sleep; $s > 7$;
14. In 1955, the minimum wage in the U.S. was $0.75 per hour.
w = wage; $w \geq 0.75$;

READING STRATEGIES

To write and graph inequalities, you must connect words with symbols. Look at the information in the table below.

Symbol	Words	Graph
$<$	"less than"	
$\leq$	"less than or equal to" "no more than" "at most"	
$>$	"greater than"	
$\geq$	"greater than or equal to" "no less than" "at least"	

Answer each question.

1. What are three ways to say $x \geq 5$ in words?
x is greater than or equal to 5
x is at least 5
x is no less than 5
2. Does the graph of $p > 8$ have an empty circle or a solid circle? Why?
empty; because the value 8 is not a solution
3. Draw the graph of $m \leq -2$.
4. Write the inequality that represents the graph, once with symbols and once with words.
$t < 6.5$ **t is less than 6.5**
5. Mitchell's goal is to get a grade g of at least 85% on his next Algebra quiz. Translate these words into symbols. Then graph the inequality.
$g \geq 85$ graph:

RETEACH

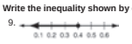
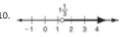
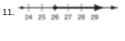
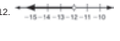
Describe the solutions of each inequality in words.

1. $t - 1 \geq 7$
2. $-6 > 2d$
3. $-4 < r + 5$
4. $\frac{1}{2}x \leq 9$

Graph each inequality.

5. $k > -12$ graph:
6. $-6\frac{1}{2} \leq w$ graph:
7. $b \leq 2^3 - 10$ graph:
8. $n < -\sqrt{2(5)+6}$ graph:

Write the inequality shown by each graph.

9. 
10. 
11. 
12. 

Define a variable and write an inequality for each situation.

13. To qualify for the job, applicants must have more than 3 years of experience in the field.
14. As of Aug. 1996, the speed limit on rural interstates in North Carolina is 70 mph.
15. In 2005, the minimum wage in the U.S. was $5.15 per hour.

56. Critical Thinking Graph all positive integer solutions of the inequality $x < 5$.

57. Write About It Explain how to write an inequality that is modeled by a graph. What characteristics do you look for in the graph?

58. Write About It You were told in the lesson that the phrase "no more than" means "less than or equal to" and the phrase "at least" means "greater than or equal to."

a. What does the phrase "at most" mean? less than or equal to

b. What does the phrase "no less than" mean? greater than or equal to

TEST PREP

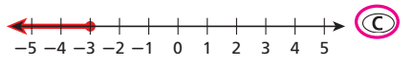
59. Which is NOT a solution of the inequality $5 - 2x \geq -3$?

(A) 0 (B) 2 (C) 4 (D) 5

60. Which is NOT a solution of the inequality $3 - x < 2$?

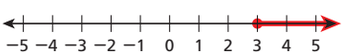
(F) 1 (G) 2 (H) 3 (J) 4

61. Which graph represents the solutions of $-2 \leq 1 - t$?

(A) 

(C)

(B)

(D) 

CHALLENGE AND EXTEND

Describe the values for x and y that make each inequality true.

62. $x + y \leq |x + y|$ all real numbers

63. $x^2 < xy$ any nonzero numbers such that x and y have the same sign and $|x| < |y|$

64. $x - y \geq y - x$ any numbers such that y is less than or equal to x

Complete each statement. Write < or >.

65. If $a > b$, then b < a.

66. If $x > y$ and $y > z$, then x > z.

67. Name a value of x that makes the statement $0.35 < x < 1.27$ true. any number between 0.35 and 1.27

68. Is $\frac{5}{6}$ a solution of $x < 1$? How many solutions of $x < 1$ are between 0 and 1? yes; infinitely many

69. Write About It Explain how to graph all the solutions of $x \neq 5$.

Draw an empty circle at 5. Then draw arrows going left and right from 5.

PROBLEM SOLVING

Write the correct answer.

1. A citizen must be at least 35 years old in order to run for the Presidency of the United States. Define a variable and write an inequality for this situation.
   a = age of person; $a \geq 35$
2. A certain elevator can hold no more than 2500 pounds. Define a variable and write an inequality for this situation.
   w = weight; $w \leq 2500$
3. Approximately 30% of the land on Earth is forested, but this percent is decreasing due to construction. Write and graph an inequality for this situation.
   f = percent forested; $f \leq 30$

4. Khalil weighed 125 pounds before he started to gain weight to play football. Write and graph an inequality for this situation.
   w = weight; $w \geq 125$

The Sanchez family is visiting an amusement park. When they enter the park, they receive a brochure which lists several requirements and restrictions. Select the best answer.

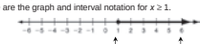
5. You must be at least 50 inches tall to ride The Wild Tornado roller coaster. Which of the following inequalities fits this situation?
   A $h \leq 50$ C $h \geq 50$
   B $h < 50$ D $h > 50$
6. Children less than 12 years old must be accompanied by an adult inside The Haunted House. Which of the following inequalities shows the ages of children who require an adult inside the house?
   F $y \leq 12$ H $y \geq 12$
   G $y < 12$ J $y > 12$
7. Totland is an area of the amusement park set aside for children who are 6 years old or younger. Which of the following inequalities represents the ages of children who are allowed in Totland?
   A $a \leq 6$ C $a \geq 6$
   B $a < 6$ D $a > 6$
8. The Bumpy Cars will not be turned on if there are 5 or more empty cars. Which of the following inequalities shows the possible numbers of empty cars if the ride is going to start?
   F $c \leq 5$ H $c \geq 5$
   G $c < 5$ J $c > 5$

CHALLENGE

When you graph the solutions of an inequality, you use a solid circle to show an endpoint that is a solution and an open circle to show an endpoint that is not. You use an arrow pointing to the right to show when solutions continue toward positive infinity and an arrow pointing to the left to show when solutions continue toward negative infinity.

Interval notation is another way to show solutions of an inequality. In interval notation, the endpoints of the solutions are explicitly written out. Brackets or parentheses indicate whether or not the endpoints themselves are solutions. If the solutions continue toward infinity, an infinity symbol is written instead. The infinity symbol is always used with a parenthesis.

Here are the graph and interval notation for $x \geq 1$.

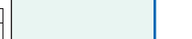
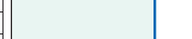
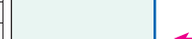
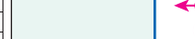
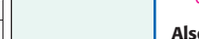
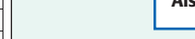


The lower endpoint is 1, and it is a solution. The solutions continue toward positive infinity.

$[1, \infty)$

Interval Notation	
[or]	endpoint is a solution
(or)	endpoint is not a solution
$(-\infty$	solutions continue toward negative infinity
$\infty)$	solutions continue toward positive infinity

Complete the table. Problems 7–10 preview compound inequalities, which you'll learn more about in Lesson 3–6.

	inequality	graph	interval notation
1.	$x > 4$		$(4, \infty)$
2.	$x \leq -2$		$(-\infty, -2]$
3.	$x < 3$		$(-\infty, 3)$
4.	$x \geq -5$		$[-5, \infty)$
5.	$x > 0$		$(0, \infty)$
6.	$x \leq -1$		$(-\infty, -1]$
7.	$-3 \leq x \leq 2$		$[-3, 2]$
8.	$0 < x \leq 3$		$(0, 3]$
9.	$-4 \leq x < 5$		$[-4, 5)$
10.	$-1 < x < 1$		$(-1, 1)$

TEST PREP DOCTOR Choices **B** and **C** in **Exercise 61** include zero as a solution. Substitute 0 for t to determine whether the inequality is true or false.

Answers

56.

57. Look for a solid or empty circle. A solid circle tells you to use ≤ or ≥, and an empty circle tells you to use < or >. Then look at the direction of the arrow. An arrow pointing left tells you to use < or ≤, and an arrow pointing right tells you to use > or ≥.

Journal

The phrase "x is not less than 7" means that x is greater than or equal to 7. The algebraic inequality is $x \geq 7$. Explain the meaning of the following statements and write an algebraic inequality for each:

1) x is not greater than 7.

2) x is not less than or equal to 7.

3) x is not greater than or equal to 7.

ALTERNATIVE ASSESSMENT

Write four inequalities using each of the following inequality symbols: <, ≤, >, ≥. Then graph each inequality.

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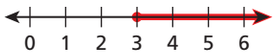
2-1 Lesson Quiz

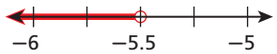
1. Describe the solutions of $7 < x + 4$. all real numbers greater than 3

2. Graph $h \geq -4.75$

Write the inequality shown by each graph.

3. 

$x \geq 3$

4. 

$x < -5.5$

5. A cell phone plan offers free minutes for no more than 250 minutes per month. Define a variable and write an inequality for the possible numbers of free minutes. Graph the solutions. $m \leq 250$ where m is nonneg.

Also available online

2-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Solve one-step inequalities by using addition.

Solve one-step inequalities by using subtraction.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Write an inequality for each situation.

1. The temperature must be at least −10°F. $x \geq -10$
2. The temperature must be no more than 90°F. $x \leq 90$

Solve each equation.

3. $x - 4 = 10$ 14
4. $15 = x + 1.1$ 13.9

Also available online

Math Humor

Q: Why did the parents think their little variable was sick?

A: The nurse said he had to be isolated.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online
State Resources Online

COMMON CORE

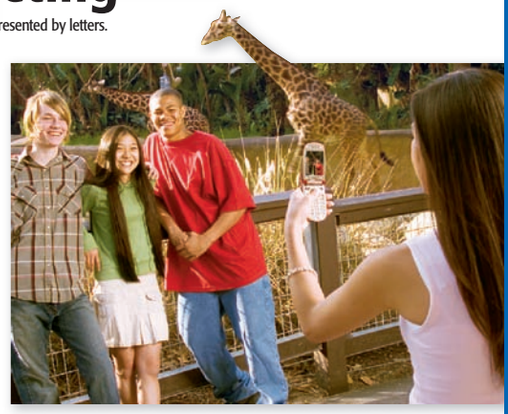
2-2 Solving Inequalities by Adding or Subtracting

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Objectives
Solve one-step inequalities by using addition.

Solve one-step inequalities by using subtraction.

Who uses this?
You can use inequalities to determine how many more photos you can take. (See Example 2.)



Tenea has a cell phone that also takes pictures. After taking some photos, Tenea can use a one-step inequality to determine how many more photos she can take.

Solving one-step inequalities is much like solving one-step equations. To solve an inequality, you need to isolate the variable using the properties of inequality and inverse operations.

Properties of Inequality

Addition and Subtraction

WORDS	NUMBERS	ALGEBRA
Addition		
You can add the same number to both sides of an inequality, and the statement will still be true.	$3 < 8$ $3 + 2 < 8 + 2$ $5 < 10$	$a < b$ $a + c < b + c$
Subtraction		
You can subtract the same number from both sides of an inequality, and the statement will still be true.	$9 < 12$ $9 - 5 < 12 - 5$ $4 < 7$	$a < b$ $a - c < b - c$

These properties are also true for inequalities that use the symbols >, ≥, and ≤.

EXAMPLE 1 Using Addition and Subtraction to Solve Inequalities

Solve each inequality and graph the solutions.

A $x + 9 < 15$

$x + 9 < 15$
$-9 \quad -9$
$x \quad < 6$

Since 9 is added to x, subtract 9 from both sides to undo the addition.

B $d - 3 > -6$

$d - 3 > -6$
$+3 \quad +3$
$d \quad > -3$

Since 3 is subtracted from d, add 3 to both sides to undo the subtraction.

Helpful Hint
Use an inverse operation to "undo" the operation in an inequality. If the inequality contains addition, use subtraction to undo the addition.

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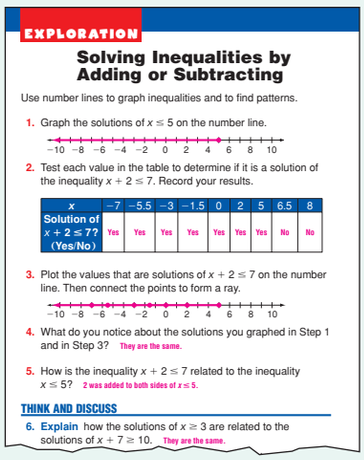
1 Introduce



EXPLORATION

Solving Inequalities by Adding or Subtracting

Use number lines to graph inequalities and to find patterns.

1. Graph the solutions of $x \leq 5$ on the number line.

2. Test each value in the table to determine if it is a solution of the inequality $x + 2 \leq 7$. Record your results.

x	−7	−5.5	−3	−1.5	0	2	5	6.5	8
Solution of $x + 2 \leq 7$? (Yes/No)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No

3. Plot the values that are solutions of $x + 2 \leq 7$ on the number line. Then connect the points to form a ray.

4. What do you notice about the solutions you graphed in Step 1 and in Step 3? They are the same.
5. How is the inequality $x + 2 \leq 7$ related to the inequality $x \leq 5$? 2 was added to both sides of $x \leq 5$.

THINK AND DISCUSS

6. **Explain** how the solutions of $x \geq 3$ are related to the solutions of $x + 7 \geq 10$. They are the same.

Motivate

Pose the following situation: Maria's mother told her that she should practice piano for at least 45 minutes. She has already practiced for 20 minutes. Ask students how they would determine the number of minutes that Maria still needs to practice. Tell students that they could use a one-step inequality to solve the problem.

Explorations and answers are provided in your online resources.

Solve each inequality and graph the solutions.

C $0.7 \geq n - 0.4$

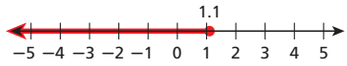
$0.7 \geq n - 0.4$

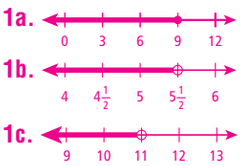
$+0.4 \quad +0.4$

$1.1 \geq n$

$n \leq 1.1$

Since 0.4 is subtracted from n, add 0.4 to both sides to undo the subtraction.





CHECK IT OUT! **Solve each inequality and graph the solutions.**

1a. $s + 1 \leq 10$ $s \leq 9$ **1b.** $2\frac{1}{2} > -3 + t$ $t < 5\frac{1}{2}$ **1c.** $q - 3.5 < 7.5$ $q < 11$

Since there can be an infinite number of solutions to an inequality, it is not possible to check all the solutions. You can check the endpoint and the direction of the inequality symbol.

The solutions of $x + 9 < 15$ are given by $x < 6$.

Step 1 Check the endpoint.

Substitute 6 for x in the related equation $x + 9 = 15$. The endpoint should be a solution of the equation.

$x + 9 = 15$	
$6 + 9$	15
15	15 ✓

Step 2 Check the inequality symbol.

Substitute a number less than 6 for x in the original inequality. The number you choose should be a solution of the inequality.

$x + 9 < 15$		
$4 + 9$	$<$	15
13	$<$	15 ✓

EXAMPLE 2 *Problem Solving Application*

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

The memory in Tenea's camera phone allows her to take up to 20 pictures. Tenea has already taken 16 pictures. Write, solve, and graph an inequality to show how many more pictures Tenea could take.

1 Understand the Problem

The **answer** will be an inequality and a graph that show all the possible numbers of pictures that Tenea can take.

List the important information:

- Tenea can take up to, or *at most*, 20 pictures.
- Tenea has taken 16 pictures already.

2 Make a Plan

Write an inequality.

Let p represent the remaining number of pictures Tenea can take.

Number taken	plus	number remaining	is at most	20 pictures.
16	+	p	≤	20

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COMMON ERROR ALERT

Students may assume that the symbol in the original inequality statement indicates the direction of the arrow in the graph. Remind students that this is only true when the variable is on the left side.

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Additional Examples

Example 1

Solve each inequality and graph the solutions.

A. $x + 12 < 20$ $x < 8$

B. $d - 5 > -7$ $d > -2$

C. $0.9 \geq n - 0.3$ $n \leq 1.2$

Example 2

Sami has a gift card. She has already used $14 of the total value, which was $30. Write, solve, and graph an inequality to show how much more she can spend. $14 + g \leq 30$; $g \leq 16$ where g is nonneg.

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- How is solving inequalities similar to solving equations?
- How are the solutions of an inequality different from the solution of an equation?

EXAMPLE 2

- What will the solutions represent?
- What two things need to be checked when checking the solutions to an inequality?

2 Teach

Guided Instruction

Discuss which operations are used to undo addition and subtraction when solving equations. Tell students to solve an inequality as they would an equation. Encourage students to always check the endpoint and the inequality symbol.

Teaching Tip **Visual** Some students may find it easier to rewrite the original inequality with the variable on the left, so that when the inequality is solved, the inequality symbol points in the direction of the shading.

Reaching All Learners *Through Critical Thinking*

Have students work in pairs. Each pair should make two sets of index cards: one set with one-step inequalities and another with the corresponding solutions and graphs. Have each pair trade cards with another pair. The students can then work together to match each inequality to its graph.

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Additional Examples

Example 3

Mrs. Lawrence wants to buy an antique bracelet at an auction. She is willing to bid no more than $550. So far, the highest bid is $475. Write and solve an inequality to determine the amount Mrs. Lawrence can add to the bid. Check your answer.

$475 + x \leq 550$; $x \leq 75$

Also available online

Questioning Strategies

EXAMPLE 3

- What words in the problem tell you which inequality symbol to use?

3 Solve

$$\begin{array}{rl} 16 + p \leq & 20 \\ -16 \quad & -16 \\ \hline p \leq & 4 \end{array}$$

Since 16 is added to p, subtract 16 from both sides to undo the addition.

It is not reasonable for Tenea to take a negative or fractional number of pictures, so graph the nonnegative integers less than or equal to 4. Tenea could take 0, 1, 2, 3, or 4 more pictures.

4 Look Back

Check Check the endpoint, 4.

$$\begin{array}{r|l} 16 + p = 20 & \\ \hline 16 + 4 & 20 \\ 20 & 20 \checkmark \end{array}$$

Check a number less than 4.

$$\begin{array}{r|c|l} 16 + p \leq 20 & & \\ \hline 16 + 2 & \leq & 20 \\ 18 & \leq & 20 \checkmark \end{array}$$

Adding 0, 1, 2, 3, or 4 more pictures will not exceed 20.

CHECK IT OUT! **2.** The Recommended Dietary Allowance (RDA) of iron for a female in Sarah's age group (14–18 years) is 15 mg per day. Sarah has consumed 11 mg of iron today. Write and solve an inequality to show how many more milligrams of iron Sarah can consume without exceeding the RDA.

$11 + m \leq 15$; $m \leq 4$ where m is nonnegative; Sarah can consume 4 mg or less without exceeding the RDA.

EXAMPLE 3 *Sports Application*

Josh can bench press 220 pounds. He wants to bench press at least 250 pounds. Write and solve an inequality to determine how many more pounds Josh must lift to reach his goal. Check your answer.

Let p represent the number of additional pounds Josh must lift.

220 pounds	plus	additional pounds	is at least	250 pounds.
220	+	p	$\geq$	250

$$\begin{array}{rl} 220 + p \geq & 250 \\ -220 \quad & -220 \\ \hline p \geq & 30 \end{array}$$

Since 220 is added to p, subtract 220 from both sides to undo the addition.

Check Check the endpoint, 30.

$$\begin{array}{r|l} 220 + p = 250 & \\ \hline 220 + 30 & 250 \\ 250 & 250 \checkmark \end{array}$$

Check a number greater than 30.

$$\begin{array}{r|c|l} 220 + p \geq 250 & & \\ \hline 220 + 40 & \geq & 250 \\ 260 & \geq & 250 \checkmark \end{array}$$

Josh must lift at least 30 additional pounds to reach his goal.

3. $250 + p > 282$; $p > 32$; Josh needs to bench press more than 32 additional pounds to break the school record.

CHECK IT OUT! **3. What if...?** Josh has reached his goal of 250 pounds and now wants to try to break the school record of 282 pounds. Write and solve an inequality to determine how many more pounds Josh needs to break the school record. Check your answer.

3 Close

Summarize

Remind students to solve one-step inequalities by using inverse operations. For the following solutions, ask students what two values of the variable they would check.

Possible answers:

$12 < c$ 12, 15

$d \geq -3$ −3, 0

$-7 > b$ −10, −7

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

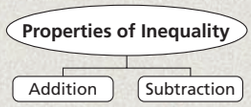
***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Show how to check your solution to Example 1B.
2. Explain how the Addition and Subtraction Properties of Inequality are like the Addition and Subtraction Properties of Equality.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an inequality that you must use the specified property to solve. Then solve and graph the inequality.

Know it! Note



2-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Solve each inequality and graph the solutions.

1. $12 < p + 6$ — $p > 6$
2. $w + 3 \geq 4$ — $w \geq 1$
3. $-5 + x \leq -20$ — $x \leq -15$
4. $z - 2 > -11$ — $z > -9$

SEE EXAMPLE 2

5. **Health** For adults, the maximum safe water temperature in a spa is 104 °F. The water temperature in Bill's spa is 102 °F. The temperature is increased by t °F. Write, solve, and graph an inequality to show the values of t for which the water temperature is still safe. $102 + t \leq 104$; $t \leq 2$ where t is nonnegative

SEE EXAMPLE 3

6. **Consumer Economics** A local restaurant will deliver food to your house if the purchase amount of your order is at least $25.00. The total for part of your order is $17.95. Write and solve an inequality to determine how much more you must spend for the restaurant to deliver your order. $17.95 + d \geq 25.00$; $d \geq 7.05$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
7–10	1
11	2
12	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each inequality and graph the solutions.

7. $a - 3 \geq 2$ — $a \geq 5$
8. $2.5 > q - 0.8$ — $q < 3.3$
9. $-45 + x < -30$ — $x < 15$
10. $r + \frac{1}{4} \leq \frac{3}{4}$ — $r \leq \frac{1}{2}$
11. **Engineering** The maximum load for a certain elevator is 2000 pounds. The total weight of the passengers on the elevator is 1400 pounds. A delivery man who weighs 243 pounds enters the elevator with a crate of weight w. Write, solve, and graph an inequality to show the values of w that will not exceed the weight limit of the elevator. $1400 + 243 + w \leq 2000$; $w \leq 357$ where w is nonnegative
12. **Transportation** The gas tank in Mindy's car holds at most 15 gallons. She has already filled the tank with 7 gallons of gas. She will continue to fill the tank with g gallons more. Write and solve an inequality that shows all values of g that Mindy can add to the car's tank. $7 + g \leq 15$; $g \leq 8$ where g is nonnegative

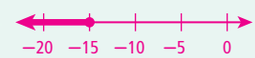
Write an inequality to represent each statement. Solve the inequality and graph the solutions.

13. Ten less than a number x is greater than 32. $x - 10 > 32$; $x > 42$
14. A number n increased by 6 is less than or equal to 4. $n + 6 \leq 4$; $n \leq -2$
15. A number r decreased by 13 is at most 15. $r - 13 \leq 15$; $r \leq 28$

Answers

1.

2.

3. 

4.

5.

7.

8.

9.

10.

11.

13.

14.

15.

Answers to *Think and Discuss*

1.

$d - 3 = -6$		$d - 3 > -6$		
$-3 - 3$	-6	$0 - 3$	$>$	-6
-6	-6 ✓	-3	$>$	-6 ✓

2. You can add or subtract the same number on both sides of an equation or an inequality, and the statement will still be true.
3. See Additional Answers.

2-2 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

Basic 7–23, 25–31, 34–39
Average 7–25, 30–42
Advanced 8–12 even, 13–25, 30–46

Homework Quick Check
Quickly check key concepts.
Exercises: 8, 10, 11, 12, 14, 24

Teaching Tip

Reading Math **Exercise 11** requires that students understand the phrases "maximum load," "not exceed," and "weight limit." Have students discuss which phrase indicates the inequality symbol for this problem.

ENGLISH LANGUAGE LEARNERS

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 5–6, 11–12, 25, 31, 35

Reason abstractly and quantitatively. Exercise 30

Construct viable arguments and critique the reasoning of others. Exercises 32–34, 46

Model with mathematics. Exercises 13–15

Look for and make use of structure. Exercise 43–45

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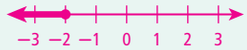
Teaching Tip **Math Background** In **Exercise 27,** students may need to be reminded that subtraction is addition of the opposite.

Multi-Step Test Prep **Exercise 35** involves using inequalities to describe the number of miles that can be driven on a trip. This exercise prepares students for the Multi-Step Test Prep.

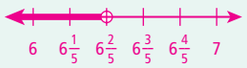
Answers

16. $x \leq -2$

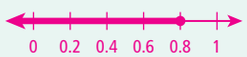


17. $q > 51$

18. $x < 6\frac{2}{5}$



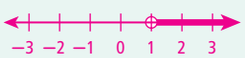
19. $p \leq 0.8$



20. $x \geq 0$

21. $c > -202$

22. $y > 1$



23. $x \geq 0$

25.

30. It is a reasonable answer. If you round each number to the nearest integer and then solve the inequality $12 + x < 22$, the solution is $x < 10$.

31. $936 + 4254 + p \leq 45{,}611$; $5190 + p \leq 45{,}611$; $p \leq 40{,}421$ where p is nonnegative

33. When you isolate the variable in each inequality, you get $x \geq 2$ and $x \geq 2$.

34–35. See Additional Answers.

PRACTICE A

PRACTICE C

PRACTICE B

Solve each inequality and graph the solutions.

1. $b + 8 > 15$ — $b > 7$

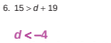
2. $t - 5 \geq -2$ — $t \geq 3$

3. $-4 + x \geq 1$ — $x \geq 5$

4. $g + 8 < 2$ — $g < -6$

5. $-9 \geq m - 9$ — $m \leq 0$

6. $15 > d + 19$ — $d < -4$



Answer each question.

7. Jessica makes overtime pay when she works more than 40 hours in a week. So far this week she has worked 29 hours. She will continue to work h hours this week. Write, solve, and graph an inequality to show the values of h that will allow Jessica to earn overtime pay. — $29 + h > 40$; $h > 11$

8. Henry's MP3 player has 512MB of memory. He has already downloaded 287MB and will continue to download m more megabytes. Write and solve an inequality that shows how many more megabytes he can download. — $287 + m \leq 512$; $m \leq 225$

9. Eleanor needs to read at least 97 pages of a book for homework. She has read 34 pages already. Write and solve an inequality that shows how many more pages p she must read. — $34 + p \geq 97$; $p \geq 63$

Solve each inequality and graph the solutions.

16. $x + 4 \leq 2$ 17. $-12 + q > 39$ 18. $x + \frac{3}{5} < 7$ 19. $4.8 \geq p + 4$

20. $-12 \leq x - 12$ 21. $4 < 206 + c$ 22. $y - \frac{1}{3} > \frac{2}{3}$ 23. $x + 1.4 \geq 1.4$

24. Use the inequality $s + 12 \geq 20$ to fill in the missing numbers.

a. $s \geq$ 8 b. $s +$ 22 ≥ 30 c. $s - 8 \geq$ 0

LINK Health

Special-effects contact lenses are sometimes part of costumes for movies. All contact lenses should be worn under an eye doctor's supervision.

25. **Health** A particular type of contact lens can be worn up to 30 days in a row. Alex has been wearing these contact lenses for 21 days. Write, solve, and graph an inequality to show how many more days Alex could wear his contact lenses.

$21 + d \leq 30$; $d \leq 9$ where d is nonnegative

Solve each inequality and match the solutions to the correct graph.

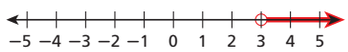
26. $1 \leq x - 2$ — $x \geq 3$; C

27. $8 > x - (-5)$ — $x < 3$; B

28. $x + 6 > 9$ — $x > 3$; A

29. $-4 \geq x - 7$ — $x \leq 3$; D

A.



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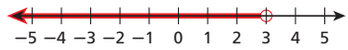

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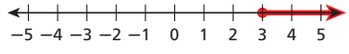
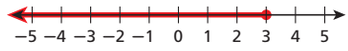


D.



30. **Estimation** Is $x < 10$ a reasonable estimate for the solutions to the inequality $11.879 + x < 21.709$? Explain your answer.

31. **Sports** At the Seattle Mariners baseball team's home games, there are 45,611 seats in the four areas listed in the table. Suppose all the suite level and club level seats during a game are filled. Write and solve an inequality to determine how many people p could be sitting in the other types of seats.

Mariners Home Game Seating	
Type of Seat	Number of Seats
Main bowl	24,399
Upper bowl	16,022
Club level	4,254
Suite level	936

32. **Critical Thinking** Recall that a balance scale was used to model solving equations. Describe how a balance scale could model solving inequalities.

32. If a scale is unbalanced and the same amount is added to or subtracted from both sides, the scale should maintain the same amount of imbalance.

33. **Critical Thinking** Explain why $x + 4 \geq 6$ and $x - 4 \geq -2$ have the same solutions.

34. **Write About It** How do the solutions of $x + 2 \geq 3$ differ from the solutions of $x + 2 > 3$? How do the graphs of the solutions differ?

Multi-Step Test Prep

35. a. Daryl finds that the distance from Columbus, Ohio, to Washington, D.C., is 411 miles. What is the round-trip distance?

b. Daryl can afford to drive a total of 1000 miles. Write an inequality to show the number of miles m he can drive while in Washington, D.C.

c. Solve the inequality and graph the solutions on a number line. Show that your answer is reasonable.

(tl), Buzz Orr/The Gazette/AP/Wide World Photos; (cr), PhotoDisc/gettyimages; (bl), © Creatas

READING STRATEGIES

There are two parts to checking solutions to inequalities: checking the endpoint and checking the direction of the inequality symbol.

Check that $x > -3$ represents the solutions to $x - 4 > -7$.

Step 1
$x - 4 = -7$ — Write the related equation.
$-3 - 4 = -7$ — Substitute -3 for x in the related equation.
$-7 = -7$ ✓ — If the statement is true, the value is an endpoint.

Step 2
$x - 4 > -7$ — Write the original inequality.
$10 - 4 > -7$ — Substitute a value greater than -3 in the original inequality.
$6 > -7$ ✓ — If the statement is true, the symbol is correct.

Both steps check, so $x > -3$ correctly represents the solutions.

Answer each question.

1. Which Step confirms that the endpoint is correct? Step 1
2. What is being checked in Step 2?
the direction of the inequality symbol
3. Describe Step 1 when checking that $8 \leq m$ represents the solutions to $14 \leq m + 6$.
Substitute 8 for m in $14 = m + 6$.
4. Give a value that could be used in Step 2 when checking that $8 \leq m$ represents the solutions to $14 \leq m + 6$.
Possible answer: 9

For each problem, check that the given solutions represent the inequality by using the two-step procedure shown above.

5. $t + 5 < -9$; $t > -14$

Step 1	Step 2	correct?
$t + 5 = -9$	$t + 5 < -9$	
$-14 + 5 = -9$	$-13 + 5 < -9$	no
$-9 = -9$	$-8 < -9$	

6. $b - 6 \geq 2$; $b \geq 8$

Step 1	Step 2	correct?
$b - 6 = 2$	$b - 6 \geq 2$	
$8 - 6 = 2$	$9 - 6 \geq 2$	yes
$2 = 2$	$3 \geq 2$	

RETEACH

The method for solving one-step inequalities by adding is just like the method for solving one-step equations by adding.

Solve $x - 2 = 1$ and graph the solution.

$x - 2 = 1$
$+2 \quad +2$ — Add 2 to each side.
$x = 3$

Solve $x - 2 \geq 1$ and graph the solutions.

$x - 2 \geq 1$
$+2 \quad +2$ — Add 2 to each side.
$x \geq 3$

Solve $-4 = a - 3$ and graph the solution.

$-4 = a - 3$
$+3 \quad +3$ — Add 3 to each side.
$-1 = a$

Solve $-4 > a - 3$ and graph the solutions.

$-4 > a - 3$
$+3 \quad +3$ — Add 3 to each side.
$-1 > a$
$a < -1$

Solve each inequality and graph the solutions.

1. $b - 4 < 3$ — $b > 7$

2. $x - 5 < -2$ — $x < 3$

3. $-10 > -6 + x$ — $x < -4$

4. $1 \leq f - 3$ — $f \geq 4$

TEST PREP

36. Which is a reasonable solution of $4.7367 + p < 20.1784$?

Ⓐ 15 Ⓑ 16 Ⓒ 24 Ⓓ 25

37. Which statement can be modeled by $x + 3 \leq 12$?

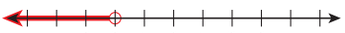
Ⓕ Sam has 3 bottles of water. Together, Sam and Dave have at most 12 bottles of water.

Ⓖ Jennie sold 3 cookbooks. To earn a prize, Jennie must sell at least 12 cookbooks.

Ⓗ Peter has 3 baseball hats. Peter and his brothers have fewer than 12 baseball hats.

Ⓙ Kathy swam 3 laps in the pool this week. She must swim more than 12 laps.

38. Which graph represents the solutions of $p + 3 < 1$?

Ⓐ 

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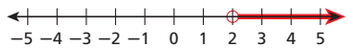
Ⓓ 

39. Which inequality does NOT have the same solutions as $n + 12 \leq 26$?

Ⓕ $n \leq 14$ Ⓖ $n + 6 \leq 20$ Ⓗ $10 \geq n - 4$ Ⓙ $n - 12 \leq 14$

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

40. $6\frac{9}{10} \geq 4\frac{4}{5} + x$ $x \leq 2\frac{1}{10}$ **41.** $r - 1\frac{2}{5} \leq 3\frac{7}{10}$ $r \leq 5\frac{1}{10}$ **42.** $6\frac{2}{3} + m > 7\frac{1}{6}$ $m > \frac{1}{2}$

Determine whether each statement is sometimes, always, or never true. Explain.

43. $a + b > a - b$
Sometimes; when b is positive, the inequality is true.

44. If $a > c$, then $a + b > c + b$.
Always; adding the same number to both sides keeps the statement true.

45. If $a > b$ and $c > d$, then $a + c > b + d$.
Always; a and c are the two greater numbers, so their sum will be greater than that of the two lesser numbers.

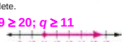
46. If $x + b > c$ and $x > 0$ have the same solutions, what is the relationship between b and c? b and c are equal.

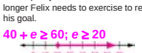
PROBLEM SOLVING

Write the correct answer.

1. Sumiko is allowed to watch no more than 10 hours of television each week. She has watched 4 hours of television already. Write and solve an inequality to show how many more hours of television Sumiko can watch.
   $4 + h \leq 10$; $h \leq 6$

2. A satellite will be released into an orbit of more than 400 miles above the Earth. The rocket carrying it is currently 255 miles above Earth. Write and solve an inequality to show how much higher the rocket must climb before it releases the satellite.
   $m + 255 > 400$; $m > 145$

3. Wayne's homework is to solve at least 20 questions from his textbook. So far, he has completed 9 of them. Write, solve, and graph an inequality to show how many more problems Wayne must complete.
   $q + 9 \geq 20$; $q \geq 11$
   

4. Felix wants to get at least one hour of exercise each day. Today, he has run for 40 minutes. Write, solve, and graph an inequality that shows how much longer Felix needs to exercise to reach his goal.
   $40 + e \geq 60$; $e \geq 20$
   

The high school has been raising money for charity and the class that raises the most will be awarded a party at the end of the year. The table below shows how much money each class has raised so far. Use this information to answer questions 5–7.

Class	Amount Raised ($)
Seniors	870
Juniors	650
Sophomores	675
First-Years	590

5. The school has a goal of raising at least $3000. Which inequality shows how much more money m they need to raise to reach their goal?
   A $m \geq 215$ C $m \leq 215$
   B $m < 215$ D $m > 2785$

6. The juniors would like to raise more money than the seniors. The seniors have completed their fundraising for the year. Which expression shows how much more money j the juniors must raise to overtake the seniors?
   F $j \leq 220$ H $j \geq 220$
   G $j < 220$ J $j > 220$

7. A local business has agreed to donate no more than half as much as the senior class raises. Which inequality shows how much money b the business will contribute?
   A $\frac{1}{2}(870) \leq b$ C $\frac{1}{2}(870) \geq b$
   B $870 \leq \frac{1}{2}b$ D $870 \geq \frac{1}{2}b$

CHALLENGE

Susan has 200 feet of fencing and wants to use it to enclose a rectangular garden. She knows that the formula for the perimeter of a rectangle is $P = 2(L + W)$. Susan also knows that the area A of a rectangle is found by using $A = LW$.

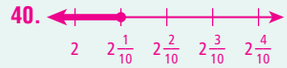
Answer the following questions in order to help Susan plan how to use the fencing.

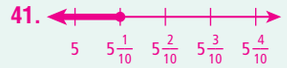
1. Susan can either use some or all of the fencing. Write an inequality that describes how much fencing she could use. $0 < P \leq 200$
2. Would a length of 70 feet and a width of 40 feet satisfy the inequality in Exercise 1? Explain.
   No; because $2(70 + 40) = 220$ feet is greater than 200 feet
3. a. If Susan decides to use all of the fencing and chooses a length of 20 feet, find the width. 80 feet
   b. What would the length be if she chooses a width of 20 feet? 80 feet
   c. Given these dimensions, find the area of the garden. 1600 square feet
4. a. If Susan decides to use all of the fencing and chooses a length of 40 feet, find the width. 60 feet
   b. What would the length be if she chooses a width of 40 feet? 60 feet
   c. Given these dimensions, find the area of the garden. 2400 square feet
5. What conclusion can you draw from Exercises 3 and 4?
   If the dimensions are reversed, the area remains the same.
6. a. If Susan would like the length and the width to be as close to one another as possible and to be whole numbers, what might the dimensions be? 50 feet by 50 feet
   b. Given her intention in part a, what shape is she trying to achieve? a square
   c. Given your answer to part a, find the area of the garden. 2500 square feet
7. Susan's friend, Jack, has 160 feet of fencing. He wants to use all his fencing to make a garden with the greatest possible area. Without using inequalities, what dimensions should he choose? 40 feet by 40 feet

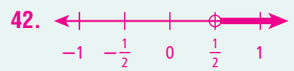
TEST PREP DOCTOR If students have difficulty selecting the correct statement in **Exercise 37,** remind them that the inequality indicates 12 is the greatest value the expression can equal.

In **Exercise 38,** students who chose **D** may not have remembered that the less-than symbol means the graph will be shaded to the left of the endpoint.

Answers

40. 

41. 

42. 

Journal

Explain why the solutions to the inequality $x + 3 \geq 12$ are not only those numbers greater than or equal to 12.

ALTERNATIVE ASSESSMENT

Have students write a real-world situation that can be modeled by a one-step inequality. Ask students to write, solve, and graph the inequality and explain the meaning of the solution in the context of the situation.

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2-2 Lesson Quiz

Solve each inequality and graph the solutions.

1. $13 < x + 7$ $x > 6$

2. $-6 + h \geq 15$ $h \geq 21$

3. $6.7 + y \leq -2.1$ $y \leq -8.8$

4. A restaurant has room for 120 customers. There are 72 customers dining. Write and solve an inequality to show how many more people can eat at the restaurant.
$x + 72 \leq 120$; $x \leq 48$ where x is a whole number

Also available online

2-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Solve one-step inequalities by using multiplication.

Solve one-step inequalities by using division.

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Warm Up

Solve each equation.

1. $-5a = 30$ -6
2. $-\frac{2}{5}x = 4$ -10
3. $\frac{y}{-5} = \frac{-3}{8}$ $1\frac{7}{8}$
4. $-\frac{3}{4}x = \frac{6}{11}$ $-\frac{8}{11}$

Graph each inequality.

5. $x \geq -10$

6. $x < -3$

Also available online

Math Humor

Q: What did the teacher do to prepare for class?

A: She made a "less-than" plan. (Lesson plan)

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

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COMMON CORE

2-3 Solving Inequalities by Multiplying or Dividing

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Also **CC.9-12.A.CED.1***

Objectives
Solve one-step inequalities by using multiplication.

Solve one-step inequalities by using division.

Who uses this?
You can solve an inequality to determine how much you can buy with a certain amount of money. (See Example 3.)

Remember, solving inequalities is similar to solving equations. To solve an inequality that contains multiplication or division, undo the operation by dividing or multiplying both sides of the inequality by the same number.

The rules below show the properties of inequality for multiplying or dividing by a positive number. The rules for multiplying or dividing by a negative number appear later in this lesson.

Jonny Hawkins/CartoonResource.com

"This is all I have, so I'll take 3 pencils, 3 notebooks, a binder, and 0.9 calculators."

Know it! Note

Properties of Inequality

Multiplication and Division by Positive Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication		
You can multiply both sides of an inequality by the same *positive* number, and the statement will still be true.	$7 < 12$ $7(3) < 12(3)$ $21 < 36$	If $a < b$ and $c > 0$, then $ac < bc$.
Division		
You can divide both sides of an inequality by the same *positive* number, and the statement will still be true.	$15 < 35$ $\frac{15}{5} < \frac{35}{5}$ $3 < 7$	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

EXAMPLE 1 **Multiplying or Dividing by a Positive Number**

Solve each inequality and graph the solutions.

A $3x > -27$

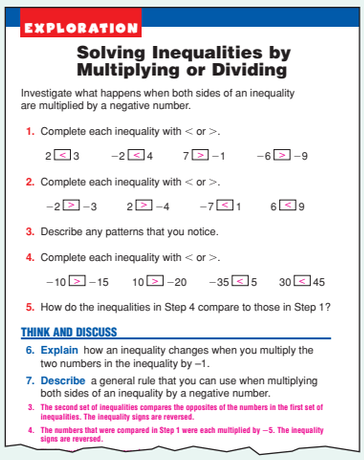
$3x > -27$

$\frac{3x}{3} > \frac{-27}{3}$

Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.

$x > -9$

1 Introduce



EXPLORATION

Solving Inequalities by Multiplying or Dividing

Investigate what happens when both sides of an inequality are multiplied by a negative number.

1. Complete each inequality with $<$ or $>$.
   $2 \boxed{<} 3$ $-2 \boxed{<} 4$ $7 \boxed{>} -1$ $-6 \boxed{>} -9$
2. Complete each inequality with $<$ or $>$.
   $-2 \boxed{>} -3$ $2 \boxed{>} -4$ $-7 \boxed{<} 1$ $6 \boxed{<} 9$
3. Describe any patterns that you notice.
4. Complete each inequality with $<$ or $>$.
   $-10 \boxed{>} -15$ $10 \boxed{>} -20$ $-35 \boxed{<} 5$ $30 \boxed{<} 45$
5. How do the inequalities in Step 4 compare to those in Step 1?

THINK AND DISCUSS

6. **Explain** how an inequality changes when you multiply the two numbers in the inequality by −1.
7. **Describe** a general rule that you can use when multiplying both sides of an inequality by a negative number.

3. The second set of inequalities compares the opposites of the numbers in the first set of inequalities. The inequality signs are reversed.
4. The numbers that were compared in Step 1 were each multiplied by −5. The inequality signs are reversed.

Motivate

Pose the following situation: "Bradley has $50 of birthday money to spend at a resale store. He wants to buy some used video games that cost $11 each." Ask students how they could determine the number of video games Bradley can buy. Tell students that they could use a one-step inequality. Tell students that there are special rules when multiplying or dividing both sides of an inequality by a negative number.

Explorations and answers are provided in your online resources.

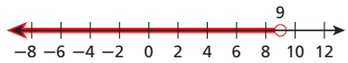
Solve each inequality and graph the solutions.

B $\frac{2}{3}r < 6$

$\frac{2}{3}r < 6$ *Since r is multiplied by $\frac{2}{3}$, multiply both sides by the reciprocal of $\frac{2}{3}$.*

$\frac{3}{2}\left(\frac{2}{3}r\right) < \frac{3}{2}(6)$

$r < 9$



1a.

1b.

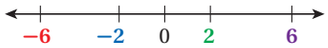
1c.

CHECK IT OUT! **Solve each inequality and graph the solutions.**

1a. $4k > 24$ $k > 6$ **1b.** $-50 \geq 5q$ $q \leq -10$ **1c.** $\frac{3}{4}g > 27$ $g > 36$

What happens when you multiply or divide both sides of an inequality by a negative number?

Look at the number line below.



$2 < 6$	$6 > -2$
$-2 \square -6$ *Multiply both sides by −1.*	$-6 \square 2$ *Multiply both sides by −1.*
$-2 > -6$ *Use the number line to determine the direction of the inequality.*	$-6 < 2$ *Use the number line to determine the direction of the inequality.*

Notice that when you multiply (or divide) both sides of an inequality by a negative number, you must reverse the inequality symbol. This means there is another set of properties of inequality for multiplying or dividing by a negative number.

Know it! Note

Properties of Inequality

Multiplication and Division by Negative Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication		
If you multiply both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$8 > 4$ $8(-2) < 4(-2)$ $-16 < -8$	If $a > b$ and $c < 0$, then $ac < bc$.
Division		
If you divide both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$12 > 4$ $\frac{12}{-4} < \frac{4}{-4}$ $-3 < -1$	If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for inequalities that use the symbols <, ≥, and ≤.

COMMON ERROR ALERT

Students may change the inequality symbol when any number in the inequality is negative. Remind them that the inequality symbol is changed only when multiplying or dividing both sides by a negative number.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Solve each inequality and graph the solutions.

A. $7x > -42$ $x > -6$

B. $2.4 \leq \frac{m}{3}$ $m \geq 7.2$

C. $\frac{3}{4}r < 12$ $r < 16$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- How do you know whether to multiply or divide?

2 Teach

Guided Instruction

Review solving one-step equations using multiplication and division. Multiply both sides of a true inequality, like $2 < 6$, by both a positive and negative number, to show that the statement is still true after it is multiplied by the positive number, but false after it is multiplied by the negative number. The false statement becomes true when the inequality sign is changed. Remind students how to check their solutions.

Reaching All Learners

Through Modeling

Draw a number line with 1 and 3 plotted.

Ask if it is correct to write $1 < 3$. yes

Multiply both numbers by -1 and plot the products.

Ask if it is correct to write $-1 < -3$. no Why? -1 is further to the right. Ask if it is correct to write $-1 > -3$. yes What changed? the direction of the sign Use similar steps to show dividing both sides of a numerical inequality by -1.

Power Presentations with PowerPoint®

Additional Examples

Example 2

Solve each inequality and graph the solutions.

A. $-12x > 84$ $x < -7$

B. $-8 \le \frac{x}{-3}$ $x \le 24$

Example 3

Jill has a $20 gift card to an art supply store where 4 oz tubes of paint are $4.30 each after tax. What are the possible numbers of tubes that Jill can buy?

0, 1, 2, 3, or 4 tubes

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 2

- What must be done to both sides of an inequality to make the inequality symbol change?

EXAMPLE 3

- Why would the solutions of the inequality be graphed as individual points?

EXAMPLE 2 Multiplying or Dividing by a Negative Number

Solve each inequality and graph the solutions.

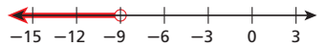
Caution!

Do not change the direction of the inequality symbol just because you see a negative sign. For example, you do not change the symbol when solving $4x < -24$.

A $-8x > 72$

$\frac{-8x}{-8} < \frac{72}{-8}$ *Since x is multiplied by −8, divide both sides by −8. Change > to <.*

$x < -9$



B $-3 \le \frac{x}{-5}$

$-5(-3) \ge -5\left(\frac{x}{-5}\right)$ *Since x is divided by −5, multiply both sides by −5. Change ≤ to ≥.*

$15 \ge x$ (or $x \le 15$)

CHECK IT OUT! **Solve each inequality and graph the solutions.**

2a. $10 \ge -x$ $x \ge -10$

2b. $4.25 > -0.25h$ $h > -17$

EXAMPLE 3 *Consumer Application*

Ryan has a $16 gift card for a health store where a smoothie costs $2.50 with tax. What are the possible numbers of smoothies that Ryan can buy?

Let s represent the number of smoothies Ryan can buy.

$2.50	times	number of smoothies	is at most	$16.00.
2.50	•	s	≤	16.00

$2.50s \le 16.00$

$\frac{2.50s}{2.50} \le \frac{16.00}{2.50}$ *Since s is multiplied by 2.50, divide both sides by 2.50. The symbol does not change.*

$s \le 6.4$ *Ryan can buy only a whole number of smoothies.*

Ryan can buy 0, 1, 2, 3, 4, 5, or 6 smoothies.

CHECK IT OUT! **3.** A pitcher holds 128 ounces of juice. What are the possible numbers of 10-ounce servings that one pitcher can fill?

$10g \le 128$; $g \le 12.8$; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 servings

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Compare the Multiplication and Division Properties of Inequality and the Multiplication and Division Properties of Equality.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each cell, write and solve an inequality.

Solving Inequalities by Using Multiplication and Division		
	By a Positive Number	By a Negative Number
Divide		
Multiply		

Sam Dudgeon/HMH

3 Close

Summarize

Remind students that solving one-step inequalities is like solving one-step equations, except they must remember to change the sign of the inequality when multiplying or dividing by a negative number. Write the following on the board, and ask if the inequality sign would change when solving.

$2x > -10$ no

$-3b \le 12$ yes

$5 > -10n$ yes

$-16 < 4t$ no

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. They are alike because you can multiply or divide both sides by a positive number and the inequality or equation will still be true. They are different because you have to reverse the inequality symbol if you multiply or divide both sides by a negative number.
2. See Additional Answers.

2-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1

1. $3b > 27$
2. $-40 \geq 8b$
3. $\frac{d}{3} > 6$
4. $24d \leq 6$
5. $1.1m \leq 1.21$
6. $\frac{2}{3}k > 6$
7. $9s > -18$
8. $\frac{4}{5} \geq \frac{r}{2}$

SEE EXAMPLE 2

9. $-2x < -10$
10. $\frac{b}{-2} \geq 8$
11. $-3.5n < 1.4$
12. $4 > -8g$
13. $\frac{d}{-6} < \frac{1}{2}$
14. $-10h \geq -6$
15. $12 > \frac{t}{-6}$
16. $-\frac{1}{2}m \geq -7$

SEE EXAMPLE 3

17. **Travel** Tom saved $550 to go on a school trip. The cost for a hotel room, including tax, is $80 per night. What are the possible numbers of nights Tom can stay at the hotel? **0, 1, 2, 3, 4, 5, or 6 nights**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–29	1
30–41	2
42	3

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each inequality and graph the solutions.

18. $10 < 2t$
19. $\frac{1}{3}j \leq 4$
20. $-80 < 8c$
21. $21 > 3d$
22. $\frac{w}{4} \geq -2$
23. $\frac{h}{4} \leq \frac{2}{7}$
24. $6y < 4.2$
25. $12c \leq -144$
26. $\frac{4}{5}x \geq \frac{2}{5}$
27. $6b \geq \frac{3}{5}$
28. $-25 > 10p$
29. $\frac{b}{8} \leq -2$
30. $-9a > 81$
31. $\frac{1}{2} < \frac{r}{-3}$
32. $-6p > 0.6$
33. $\frac{y}{-4} > -\frac{1}{2}$
34. $-\frac{1}{6}f < 5$
35. $-2.25t < -9$
36. $24 \leq -10w$
37. $-11z > 121$
38. $\frac{3}{5} < \frac{f}{-5}$
39. $-k \geq 7$
40. $-2.2b < -7.7$
41. $16 \geq -\frac{4}{3}p$

42. **Camping** The rope Roz brought with her camping gear is 54 inches long. Roz needs to cut shorter pieces of rope that are each 18 inches long. What are the possible number of pieces Roz can cut? **0, 1, 2, or 3**

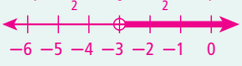
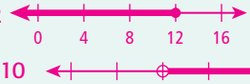
Solve each inequality and graph the solutions.

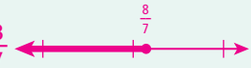
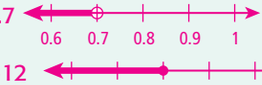
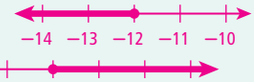
43. $-8x < 24$
44. $3t \leq 24$
45. $\frac{1}{4}x < 5$
46. $\frac{4}{5}p \geq -24$
47. $54 \leq -9p$
48. $3t > -\frac{1}{2}$
49. $-\frac{3}{4}b > -\frac{3}{2}$
50. $216 > 3.6r$

55. You reverse the symbol only when you multiply or divide both sides of the inequality by the same negative number.

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. The product of a number and 7 is not less than 21. **$7x \geq 21$; $x \geq 3$**
52. The quotient of h and -6 is at least 5. **$\frac{h}{-6} \geq 5$; $h \leq -30$**
53. The product of $-\frac{4}{5}$ and b is at most -16. **$-\frac{4}{5}b \leq -16$; $b \geq 20$**
54. Ten is no more than the quotient of t and 4. **$10 \leq \frac{t}{4}$; $t \geq 40$**
55. **Write About It** Explain how you know whether to reverse the inequality symbol when solving an inequality.
56. **Geometry** The area of a rectangle is at most 21 square inches. The width of the rectangle is 3.5 inches. What are the possible measurements for the length of the rectangle? **$\ell \leq 6$ in. where ℓ is positive**

12. $g > -\frac{1}{2}$
13. $d > -3$ 
14. $h \leq 0.6$
15. $t > -72$
16. $m \leq 14$
18. $t > 5$
19. $j \leq 12$ 
20. $c > -10$

21. $d < 7$
22. $w \geq -8$
23. $h \leq \frac{8}{7}$ 
24. $y < 0.7$ 
25. $c \leq -12$ 
26. $x \geq \frac{1}{2}$

27–41, 43–50. See Additional Answers.
51–54. For graphs, see Additional Answers.

Assignment Guide

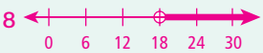
Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic 18–60, 64–70
Average 18–56, 61–70, 72
Advanced 18–42, 44–50 even, 51–56, 61, 63, 66–77

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 34, 42, 50, 54, 62

Answers

1. $b > 9$
2. $b \leq -5$
3. $d > 18$ 
4. $d \leq \frac{1}{4}$
5. $m \leq 1.1$
6. $k > 9$
7. $s > -2$
8. $r \leq \frac{8}{5}$
9. $x > 5$
10. $b \leq -16$
11. $n > -0.4$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 17, 42, 56, 61–62, 65–66

Reason abstractly and quantitatively. Exercise 75

Construct viable arguments and critique the reasoning of others. Exercises 55, 63–64, 70, 76–77

Model with mathematics. Exercises 51–54

Learn It Online
State Resources Online

Teaching Tip **Inclusion** Some students may get confused in **Exercise 59** because the inequality changes direction twice: once when dividing both sides by a negative number, and again when rewriting the expression with the variable on the left. Encourage students to write down every step, and not to attempt to do two steps at once.

MULTI-STEP TEST PREP **Exercise 66** involves writing and solving an inequality in which the reasonable answers are restricted to natural numbers. This exercise prepares students for the Multi-Step Test Prep.

Answers

66b.

Solve each inequality and match the solution to the correct graph.

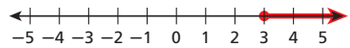
57. $-0.5t \geq 1.5$ $t \leq -3$; C

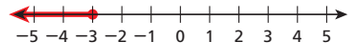
58. $\frac{1}{9}t \leq -3$ $t \leq -27$; D

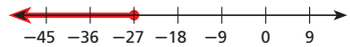
59. $-13.5 \leq -4.5t$ $t \leq 3$; A

60. $\frac{t}{-6} \leq -\frac{1}{2}$ $t \geq 3$; B

A.

B. 

C. 

D. 

LINK **Animals**

Orangutans weigh about 3.5 pounds at birth. As adults, female orangutans can weigh as much as 110 pounds, and male orangutans can weigh up to 300 pounds.

61. **Animals** A wildlife shelter is home to birds, mammals, and reptiles. If cat chow is sold in 20 lb bags, what is the least number of bags of cat chow needed for one year at this shelter? 26 bags

Food Consumed at a Wildlife Shelter per Week	
Type of Food	Amount of Food (lb)
Grapes	4
Mixed seed	10
Peanuts	5
Cat chow	10
Kitten chow	5

62. **Education** In order to earn an A in a college math class, a student must score no less than 90% of all possible points. One semester, a student with 567 points earned an A in the class. Write an inequality to show the numbers of points possible. $p \leq 630$ where p is a whole number

63. **Critical Thinking** Explain why you cannot solve an inequality by multiplying both sides by zero. Multiplying both sides of an inequality by zero makes both sides equal zero, so there is no longer an inequality to solve.

64. ///ERROR ANALYSIS/// Two students have different answers for a homework problem. Which answer is incorrect? Explain the error.

A is incorrect. Both sides are divided by a positive number, so the inequality symbol should not be reversed.

A
$9m \geq -27$
$\frac{9m}{9} \geq \frac{-27}{9}$
$m \leq -3$

B
$9m \geq -27$
$\frac{9m}{9} \geq \frac{-27}{9}$
$m \geq -3$

65. Jan has a budget of $800 for catering. The catering company charges $12.50 per guest. Write and solve an inequality to show the numbers of guests Jan can invite. $12.5g \leq 800$; $g \leq 64$ where g is a whole number

MULTI-STEP TEST PREP

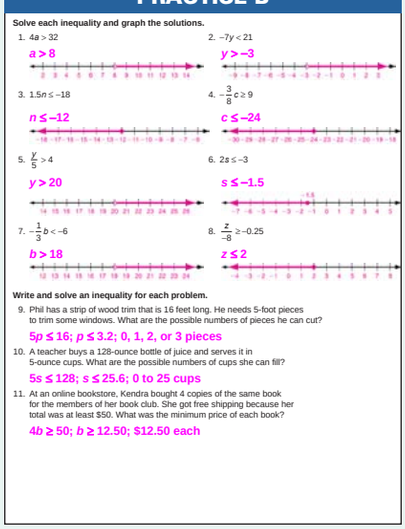
66. a. The Swimming Club can spend a total of $250 for hotel rooms for its spring trip. One hotel costs $75 per night. Write an inequality to find the number of rooms the club can reserve at this hotel. Let n be the number of rooms. $75n \leq 250$

b. Solve the inequality you wrote in part **a.** Graph the solutions on a number line. Make sure your answer is reasonable. $n \leq 3.33$; n must be a natural number.

c. Another hotel offers a rate of $65 per night. Does this allow the club to reserve more rooms? Explain your reasoning.
$65n \leq 250$; $n \leq 3.85$; no, they still can reserve only 3 or fewer rooms.

(cl), © Reuters/CORBIS; (bl), © Creatas

PRACTICE A
PRACTICE C
PRACTICE B



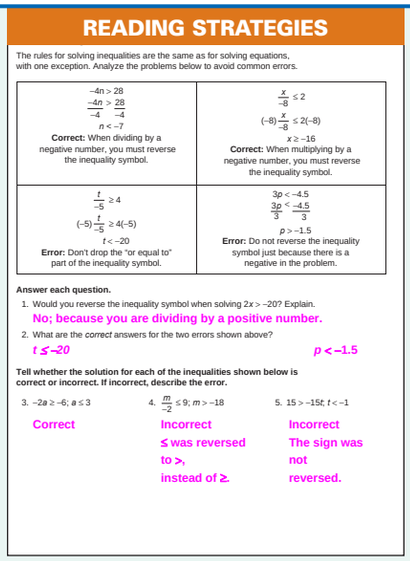
Solve each inequality and graph the solutions.

1. $4a > 32$ $a > 8$
2. $-7y < 21$ $y > -3$
3. $1.5n \leq -18$ $n \leq -12$
4. $-\frac{3}{8}c \geq 9$ $c \leq -24$
5. $\frac{y}{5} > 4$ $y > 20$
6. $2s \leq -3$ $s \leq -1.5$
7. $-\frac{1}{3}b < -6$ $b > 18$
8. $\frac{z}{-8} \geq -0.25$ $z \leq 2$

Write and solve an inequality for each problem.

9. Phil has a strip of wood trim that is 16 feet long. He needs 5-foot pieces to trim some windows. What are the possible numbers of pieces he can cut? $5p \leq 16$; $p \leq 3.2$; 0, 1, 2, or 3 pieces
10. A teacher buys a 128-ounce bottle of juice and serves it in 5-ounce cups. What are the possible numbers of cups she can fill? $5s \leq 128$; $s \leq 25.6$; 0 to 25 cups
11. At an online bookstore, Kendra bought 4 copies of the same book for the members of her book club. She got free shipping because her total was at least $50. What was the minimum price of each book? $4b \geq 50$; $b \geq 12.50$; $12.50 each

READING STRATEGIES



The rules for solving inequalities are the same as for solving equations, with one exception. Analyze the problems below to avoid common errors.

$\frac{-4n}{-4} > \frac{28}{-4}$ $n < -7$ **Correct:** When dividing by a negative number, you must reverse the inequality symbol.	$\frac{x}{-8} \leq 2$ $(-8)\frac{x}{-8} \leq 2(-8)$ $x \geq -16$ **Correct:** When multiplying by a negative number, you must reverse the inequality symbol.
$\frac{t}{-5} \geq 4$ $(-5)\frac{t}{-5} \geq 4(-5)$ $t < -20$ **Error:** Don't drop the "or equal to" part of the inequality symbol.	$3p < -4.5$ $\frac{3p}{3} < \frac{-4.5}{3}$ $p > -1.5$ **Error:** Do not reverse the inequality symbol just because there is a negative in the problem.

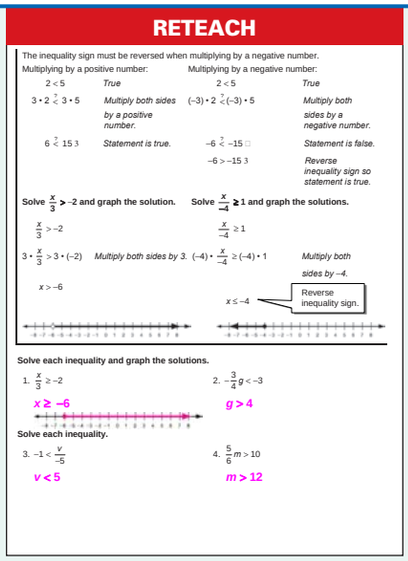
Answer each question.

1. Would you reverse the inequality symbol when solving $2x > -20$? Explain. No; because you are dividing by a positive number.
2. What are the *correct* answers for the two errors shown above? $t \leq -20$ $p < -1.5$

Tell whether the solution for each of the inequalities shown below is correct or incorrect. If incorrect, describe the error.

3. $-2a \geq -6$; $a \leq 3$ Correct
4. $\frac{m}{-2} \leq 9$; $m > -18$ Incorrect ≤ was reversed to >, instead of ≥.
5. $15 > -15t$; $t < -1$ Incorrect The sign was not reversed.

RETEACH



The inequality sign must be reversed when multiplying by a negative number.

Multiplying by a positive number:
$2 < 5$ True
$3 \cdot 2 \stackrel{?}{<} 3 \cdot 5$ Multiply both sides by a positive number.
$6 \stackrel{?}{<} 15$ ✓ Statement is true.

Multiplying by a negative number:
$2 < 5$ True
$(-3) \cdot 2 \stackrel{?}{<} (-3) \cdot 5$ Multiply both sides by a negative number.
$-6 \stackrel{?}{<} -15$ □ Statement is false.
$-6 > -15$ ✓ Reverse inequality sign so statement is true.

Solve $\frac{x}{3} > -2$ and graph the solution.
$\frac{x}{3} > -2$
$3 \cdot \frac{x}{3} > 3 \cdot (-2)$ Multiply both sides by 3.
$x > -6$

Solve $\frac{x}{-4} \geq 1$ and graph the solutions.
$\frac{x}{-4} \geq 1$
$(-4) \cdot \frac{x}{-4} \geq (-4) \cdot 1$ Multiply both sides by −4.
$x \leq -4$ Reverse inequality sign.

Solve each inequality and graph the solutions.

1. $\frac{x}{3} \geq -2$ $x \geq -6$
2. $-\frac{3}{4}g < -3$ $g > 4$

Solve each inequality.

3. $-1 < \frac{v}{-5}$ $v < 5$
4. $\frac{5}{6}m > 10$ $m > 12$

TEST PREP

67. Which inequality does NOT have the same solutions as $-\frac{2}{3}y > 4$?

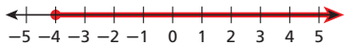
Ⓐ $12 < -2y$

Ⓑ $\frac{y}{2} < -12$

Ⓒ $-\frac{3}{4}y > \frac{9}{2}$

Ⓓ $-3y > 18$

68. The solutions of which inequality are NOT represented by the following graph?



Ⓕ $\frac{x}{2} \geq -2$

Ⓖ $-5x \geq 20$

Ⓗ $3x \geq -12$

Ⓙ $-7x \leq 28$

69. Which inequality can be used to find the number of 39-cent stamps you can purchase for $4.00?

Ⓐ $0.39s \geq 4.00$

Ⓑ $0.39s \leq 4.00$

Ⓒ $\frac{s}{0.39} \leq 4.00$

Ⓓ $\frac{4.00}{0.39} \leq s$

70. Short Response Write three different inequalities that have the same solutions as $x > 4$. Show your work and explain each step.

Possible answer: $2x > 8$, $\frac{1}{4}x > 1$, $3x > 12$

CHALLENGE AND EXTEND

Solve each inequality.

71. $2\frac{1}{3} \leq -\frac{5}{6}g$ **72.** $\frac{2x}{3} < 8.25$ **73.** $2\frac{5}{8}m > \frac{7}{10}$ **74.** $3\frac{3}{5}f \geq 14\frac{2}{5}$

71. $g \leq -\frac{14}{5}$
72. $x < 12.375$
73. $m > \frac{4}{15}$
74. $f \geq 4$

75. Estimation What is the greatest possible integer solution of the inequality $3.806x < 19.902$? $x = 5$

76. Critical Thinking The Transitive Property of Equality states that if $a = b$ and $b = c$, then $a = c$. Is there a Transitive Property of Inequality using the symbol $<$? Give an example to support your answer. yes; $2 < 3$ and $3 < 4 \rightarrow 2 < 4$

77. Critical Thinking The Symmetric Property of Equality states that if $a = b$, then $b = a$. Is there a Symmetric Property of Inequality? Give an example to support your answer. no; $0 < 1$ but $1 \not< 0$

PROBLEM SOLVING

Write and solve an inequality for each situation.

1. Karin has $3 to spend in the arcade. The game she likes costs 50¢ per play. What are the possible numbers of times that she can play?

   $0.50g \leq 3$; $g \leq 6$
   0, 1, 2, 3, 4, 5, or 6

2. Tyrone has $21 and wants to buy juice drinks for his soccer team. There are 15 players on his team. How much can each drink cost so that Tyrone can buy one drink for each person?

   $15d \leq 21$; $d \leq 1.40$;
   up to $1.40

3. A swimming pool is 7 feet deep and is being filled at the rate of 2.5 feet per hour. How long can the pool be left unattended without the water overflowing?

   $2.5h \leq 7$; $h \leq 2.8$;
   up to 2.8 hours

4. Megan is making quilts that require 11 feet of cloth each. She has 50 feet of cloth. What are the possible numbers of quilts that she can make?

   $11q \leq 50$; $q \leq 4.54$;
   0, 1, 2, 3, or 4

Alyssa, Reggie, and Cassie are meeting some friends at the movies and have stopped at the refreshment stand. The table below shows some of the items for sale and their prices. Use this information to answer questions 5–7.

Menu Item	Price($)
Popcorn	3.50
Drink	3.00
Hot Dog	2.50
Nachos	2.50
Fruit Snack	2.00

5. Alyssa has $7 and would like to buy fruit snacks for as many of her friends as possible. Which inequality below can be solved to find the number of fruit snacks f she can buy?

   A $2f \leq 7$ C $7f \leq 2$
   B $2f < 7$ D $7f < 2$

6. Reggie brought $13 and is going to buy popcorn for the group. Which answer below shows the possible numbers of popcorns p Reggie can buy for his friends?

   F 0, 1, or 2 H 0, 1, 2, 3, or 4
   G 0, 1, 2, or 3 J 0, 1, 2, 3, 4, or 5

7. The movie theater donates 12% of its sales to charity. From Cassie's purchases, the theater will donate at least $2.15. Which inequality below shows the amount of money m that Cassie spent at the refreshment stand?

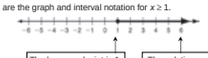
   A $m \geq 17.92$ C $m \geq 25.80$
   B $m \leq 17.92$ D $m \leq 25.80$

CHALLENGE

When you graph the solutions of an inequality, you use a solid circle to show an endpoint that is a solution and an open circle to show an endpoint that is not. You use an arrow pointing to the right to show when solutions continue toward positive infinity and an arrow pointing to the left to show when solutions continue toward negative infinity.

Interval notation is another way to show solutions of an inequality. In interval notation, the endpoints of the solutions are explicitly written out. Brackets or parentheses indicate whether or not the endpoints themselves are solutions. If the solutions continue toward infinity, an infinity symbol is written instead. The infinity symbol is always used with a parenthesis.

Here are the graph and interval notation for $x \geq 1$.



The lower endpoint is 1, and it is a solution.

The solutions continue toward positive infinity.

$[1, \infty)$

Interval Notation	
[or]	endpoint is a solution
(or)	endpoint is not a solution
$(-\infty$	solutions continue toward negative infinity
$\infty)$	solutions continue toward positive infinity

Complete the table. Problems 7–10 preview compound inequalities, which you'll learn more about in Lesson 3–6.

	inequality	graph	interval notation
1.	$x > 4$		(4, ∞)
2.	$x \leq -2$		(−∞, −2]
3.	$x < 3$		(−∞, 3)
4.	$x \geq -5$		[−5, ∞)
5.	$x > 0$		(0, ∞)
6.	$x \leq -1$		(−∞, −1]
7.	$-3 \leq x \leq 2$		[−3, 2]
8.	$0 < x \leq 3$		(0, 3]
9.	$-4 \leq x < 5$		[−4, 5)
10.	$-1 < x < 1$		(−1, 1)

TEST PREP DOCTOR

In **Exercise 69,** students who chose **A** or **D** may understand that the cost per stamp is multiplied by the number of stamps, but not that the cost must be less than or equal to $4. Students who chose **C** may not understand the relationship between the cost of the stamps and the number of stamps that can be purchased.

Journal

Have students explain why someone might change the direction of the inequality symbol twice when solving a one-step inequality with multiplication or division.

ALTERNATIVE ASSESSMENT

Have students work in groups to explain solving one-step inequalities. The report should include using each property of inequality, at least one real-world problem, and an explanation of why and when the inequality symbol is changed.

Power Presentations with PowerPoint®

2-3 Lesson Quiz

Solve each inequality and graph the solutions.

1. $8x < -24$ $x < -3$

2. $-5x \geq 30$ $x \leq -6$

3. $\frac{x}{5} > 4$ $x > 20$

4. $\frac{x}{-8} \leq \frac{-3}{4}$ $x \geq 6$

5. A soccer coach plans to order more shirts for her team. Each shirt costs $9.85. She has $77 left in her uniform budget. What are the possible numbers of shirts she can buy?
0, 1, 2, 3, 4, 5, 6, or 7 shirts

Also available online

SECTION 2A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

Answers

1. 615 mi
2. $532 + m \leq 615$; $m \leq 83$ where m is nonnegative

3. $58n \leq 200$; $n \leq 3.45$; they can reserve a maximum of 3 rooms.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Simple Inequalities

Remember the Alamo! The Spanish Club is planning a trip for next summer. They plan to travel from Fort Worth, Texas, to San Antonio, Texas. They can spend only $550 for the entire trip.

1. The treasurer of the club budgets $60 for gasoline. The current gas price is $1.95/gallon. The school van gets an average of 20 miles per gallon of gasoline. Determine how many miles they can drive on this budget. Round your answer to the nearest mile.
2. The distance from Fort Worth to San Antonio is 266 miles. Write an inequality that can be used to solve for the number of miles m that they can drive while in San Antonio. Solve your inequality and graph the solutions.
3. The treasurer budgeted $200 for hotel rooms for one night. The club chose a hotel that charges $58 per night. Write an inequality that can be used to solve for the number of rooms they can reserve n. What is the maximum number of rooms that they can reserve in the hotel?
4. Use the maximum number of rooms you found in part **3.** How much will the club spend on hotel rooms? $58(3) = \$174$
5. The club members plan to spend $80 on food. They also want to see attractions in San Antonio, such as SeaWorld and the Alamo.

   Write an inequality that can be solved to find the amount of money available for seeing attractions. What is the maximum amount the club can spend seeing attractions? $314 + a \leq 550$; $a \leq 236$
6. Write a summary of the budget for the Spanish Club trip. Include the amount they plan to spend on gasoline, hotel rooms, food, and attractions.

Gasoline	$60
Hotel rooms	$174
Food	$80
Attractions	+ $236
	$550

Sara Karen Aimee Janet Nancy Jaime Lori Ron Diana

(t), © Creatas; (br), Scott Vallance/VIP Photographic/HMH

INTERVENTION

Scaffolding Questions

1. What equation relates the number of gallons, price per gallon, and total price? total price = (price per gallon)(gallons)
2. How can you find the total distance for the round trip? Double the distance between the cities.
3. Should you round your answer up or down? Round down, because they don't have enough money for 4 rooms.
4. What operation should you use to find the cost of the hotel rooms? multiplication

5–6. What values do you need to find the amount left for seeing attractions? total budgeted amount, cost of gas, cost of hotel rooms, and cost of food

Extension

Describe any adjustments you would make to the budget. Possible answer: Transfer money from the hotel room budget to food or attractions. Students may want to make other changes.

READY TO GO ON?

Quiz for Lessons 2-1 Through 2-3

2-1 Graphing and Writing Inequalities

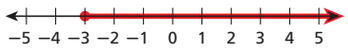
Describe the solutions of each inequality in words.

1. $-2 < r$
2. $t - 1 \leq 7$
3. $2s \geq 6$
4. $4 > 5 - x$

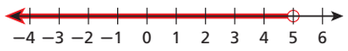
Graph each inequality.

5. $x > -2$
6. $m \leq 1\frac{1}{2}$
7. $g < \sqrt{8+1}$
8. $h \geq 2^3$

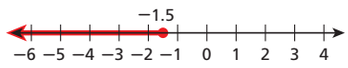
Write the inequality shown by each graph.

9. 

$x \geq -3$

10. 

$y < 5$

11. 

$z \leq -1.5$

5.

6.

7.

8.

Write an inequality for each situation and graph the solutions.

12. You must purchase at least 5 tickets to receive a discount.
13. Children under 13 are not admitted to certain movies without an adult.
14. A cell phone plan allows up to 250 free minutes per month.

2-2 Solving One-Step Inequalities by Adding or Subtracting

Solve each inequality and graph the solutions.

15. $k + 5 \leq 7$
16. $4 > p - 3$
17. $r - 8 \geq -12$
18. $-3 + p < -6$

19. Allie must sell at least 50 gift baskets for the band fund-raiser. She already sold 36 baskets. Write and solve an inequality to determine how many more baskets Allie must sell for the fund-raiser. $36 + b \geq 50$; $b \geq 14$; Allie must sell at least 14 more baskets.
20. Dante has at most \$12 to spend on entertainment each week. So far this week, he spent \$7.50. Write and solve an inequality to determine how much money Dante can spend on entertainment the rest of the week.
$7.50 + m \leq 12$; $m \leq 4.50$; Dante can spend \$4.50 at most.

2-3 Solving One-Step Inequalities by Multiplying or Dividing

Solve each inequality and graph the solutions.

21. $-4x < 8$ $x > -2$
22. $\frac{d}{3} \geq -3$ $d \geq -9$
23. $\frac{3}{4}t \leq 12$ $t \leq 16$
24. $8 > -16c$ $c > -\frac{1}{2}$

25. A spool of ribbon is 80 inches long. Riley needs to cut strips of ribbon that are 14 inches long. What are the possible numbers of strips that Riley can cut? 0, 1, 2, 3, 4, or 5

READY TO GO ON? Organizer

SECTION 2A

Objective: Assess students' mastery of concepts and skills in this section.

Resources

- *Assessment Resources* Section Quiz

INTERVENTION Resources

- *Ready to Go On? Intervention and Enrichment* Worksheets
- *Ready to Go On?* Online

Answers

1–4, 12–18. See Additional Answers.

21–24. For graphs, see Additional Answers.

READY TO GO ON? Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 2A		
	Worksheets	Online	
Lesson 2-1	2-1 Intervention	Activity 2-1	Diagnose and Prescribe Online
Lesson 2-2	2-2 Intervention	Activity 2-2	
Lesson 2-3	2-3 Intervention	Activity 2-3	

Ready to Go On? Enrichment, Section 2A
Worksheets
Online

SECTION 2B

Multi-Step and Compound Inequalities

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 2-4 Solving Two-Step and Multi-Step Inequalities • Solve inequalities that contain more than one operation. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Lesson 2-5 Solving Inequalities with Variables on Both Sides • Solve inequalities that contain variable terms on both sides. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab ***Technology Lab Activities*** Technology Lab	**Optional** graphing calculator
Algebra Lab Truth Tables and Compound Statements • Determine whether compound statements are true or false. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	
Lesson 2-6 Solving Compound Inequalities • Solve compound inequalities with one variable. • Graph solution sets of compound inequalities with one variable. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** blank transparency, yellow and blue transparency markers, graphing calculator
Lesson 2-7 Solving Absolute-Value Inequalities • Solve inequalities with one variable involving absolute-value expressions. ☑ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61a, 61b, 61c, 65, 67, 69, 71, 73, 75

Lesson 5: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51a, 51b, 51c, 53, 55, 59, 61, 63, 67, 69

Lesson 6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29a, 29b, 29c, 31, 33, 35, 37, 39, 41, 43, 47, 49, 51, 53, 55

Lesson 7: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49a, 49b, 51, 53, 55, 57

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

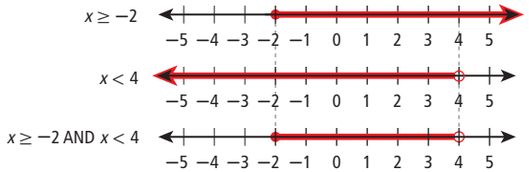
COMMON CORE PROFESSIONAL DEVELOPMENT

COMPOUND INEQUALITIES

Lesson 2-6

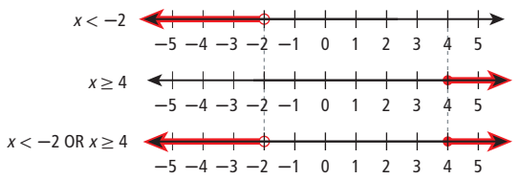
A *compound inequality* is a pair of inequalities linked by the word AND or by the word OR. To understand compound inequalities, it is important to understand the logic behind the words AND and OR and to recognize the uses of these words in basic set theory.

Consider the compound inequality $x \geq -2$ AND $x < 4$. If a value of x is a solution of the compound inequality, the word AND means that both inequalities must be true. The first two number lines below show solutions of the two inequalities; the third number line shows the points that both graphs have in common.



Thus, the solution set of the compound inequality $x \geq -2$ AND $x < 4$ is the *intersection* of the solution sets of $x \geq -2$ and $x < 4$. Graphically, this intersection is a line segment without one of its endpoints. Note that compound inequalities involving AND can often be written in a shorthand form: $-2 \leq x < 4$.

Now consider the compound inequality $x < -2$ OR $x \geq 4$. If a value of x is a solution of this compound inequality, the word OR means that x must be a solution of at least one of the two inequalities. Here the solution set is the *union* of the solution sets of $x < -2$ and $x \geq 4$. This is shown in the third number line below.



When working with compound inequalities that involve OR, students may be tempted to combine the two inequalities in a single statement, such as $-2 > x \geq 4$. This is a logical error. (For instance, the statement asserts that $-2 > 4$, which is false.) Many compound inequalities involving AND can be written in this shorthand form, but inequalities involving OR cannot.

ABSOLUTE-VALUE INEQUALITIES

Lesson 2-7

Absolute-value inequalities may be written using compound inequalities. Since the absolute value of x gives the distance of x from 0 on a number line, an inequality such as $|x| < 3$ represents all points less than 3 units from 0; that is, $x > -3$ AND $x < 3$, or $-3 < x < 3$. Similarly, $|x| > 3$ is the set of all points more than 3 units from 0, which is the compound inequality $x < -3$ OR $x > 3$.

One of the key ideas of Lesson 2-7 is that absolute-value inequalities may be used to express a range of values. The absolute value of $x - a$ gives the distance between x and a on a number line. Therefore, the absolute-value inequality $|x - a| < k$ states that the distance between x and a is less than k. In other words, the inequality represents the set of all values of x that are less than k units from a.

This idea is useful in a variety of real-world applications. For example, suppose that a machine is set to cut pieces of fabric that are 2 meters long. For quality-assurance purposes, the pieces of fabric may differ from this length by at most 3 millimeters or 0.003 meter. The range of acceptable lengths is expressed by the inequality $|x - 2| \leq 0.003$. That is, the distance on a number line from x to 2 is at most 0.003. Equivalently, the relationship can be expressed with the compound inequality $-0.003 \leq x - 2 \leq 0.003$. By the Addition Property of Inequality, it is permissible to add 2 to each part of the inequality, which gives the range of acceptable lengths as $1.997 \leq x \leq 2.003$.

2-4 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve inequalities that contain more than one operation.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each equation.

1. $2x - 5 = -17$ -6

2. $\frac{2x - 4}{3} = 8$ 14

Solve each inequality and graph the solutions.

3. $5 < t + 9$ $t > -4$

4. $\frac{a}{-2} \geq 4$ $a \leq -8$

Also available online

Math Humor

Q: What did the doctor say to the multi-step inequality?

A: I can solve your problem with a few operations.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

Learn It Online
State Resources Online

COMMON CORE

2-4 Solving Two-Step and Multi-Step Inequalities

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Also **CC.9-12.A.CED.1***

Objective
Solve inequalities that contain more than one operation.

Who uses this?
Contestants at a county fair can solve an inequality to find how many pounds a prize-winning pumpkin must weigh. (See Example 3.)

At the county fair, contestants can enter contests that judge animals, recipes, crops, art projects, and more. Sometimes an average score or average weight is used to determine the winner of the blue ribbon. A contestant can use a multi-step inequality to determine what score or weight is needed in order to win.

Inequalities that contain more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.

EXAMPLE 1 Solving Multi-Step Inequalities

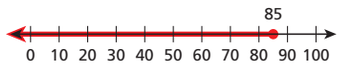
Solve each inequality and graph the solutions.

A $160 + 4f \leq 500$

$160 + 4f \leq 500$
$-160 \quad -160$
$4f \leq 340$
$\frac{4f}{4} \leq \frac{340}{4}$
$f \leq 85$

Since 160 is added to 4f, subtract 160 from both sides to undo the addition.

Since f is multiplied by 4, divide both sides by 4 to undo the multiplication.

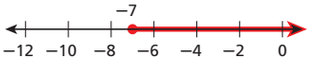


B $7 - 2t \leq 21$

$7 - 2t \leq 21$
$-7 \quad -7$
$-2t \leq 14$
$\frac{-2t}{-2} \geq \frac{14}{-2}$
$t \geq -7$

Since 7 is added to −2t, subtract 7 from both sides to undo the addition.

Since t is multiplied by −2, divide both sides by −2 to undo the multiplication. Change ≤ to ≥.



1a.

1b.

1c.

CHECK IT OUT!

Solve each inequality and graph the solutions.

1a. $-12 \geq 3x + 6$ $x \leq -6$

1b. $\frac{x + 5}{-2} > 3$ $x < -11$

1c. $\frac{1 - 2n}{3} \geq 7$ $n \leq -10$

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1 Introduce

EXPLORATION

Solving Two-Step and Multi-Step Inequalities

Explore the similarities and differences between solving equations and inequalities.

1. Consider the following steps in solving the equation $\frac{5x - 1}{2} = 7$. Fill in the rectangles to explain what was done at each step.

$\frac{5x - 1}{2} = 7$ — Multiply both sides by 2.
$5x - 1 = 14$ — Add 1 to both sides.
$5x = 15$ — Divide both sides by 5.
$x = 3$

2. Use the same steps to solve the inequality $\frac{5x - 1}{2} > 7$.
3. How is the solution of the inequality $\frac{5x - 1}{2} > 7$ different from the solution of the equation $\frac{5x - 1}{2} = 7$?
4. What would you need to do differently if the inequality was $\frac{5x - 1}{-2} > 7$?

THINK AND DISCUSS

5. **Explain** how solving an inequality is similar to solving an equation.
6. **Explain** how solving an inequality is different from solving an equation.

2. $\frac{5x - 1}{2} > 7$; $5x - 1 > 14$; $5x > 15$; $x > 3$
3. The solution of the inequality is an infinite set of numbers; the solution of the equation is a single number.

Motivate

An online movie-rental company has two movie-rental plans. Plan A costs $10 per month plus $3 for each movie rental. Plan B costs $20 per month for unlimited movie rentals. Discuss with students how to determine the number of movie rentals in one month for which plan A is less than plan B.

Tell students that they could use a two-step inequality, which they will learn to solve in this lesson.

Explorations and answers are provided in your online resources.

To solve more complicated inequalities, you may first need to simplify the expressions on one or both sides by using the order of operations, combining like terms, or using the Distributive Property.

EXAMPLE 2 Simplifying Before Solving Inequalities

Solve each inequality and graph the solutions.

A $-4 + (-8) < -5c - 2$

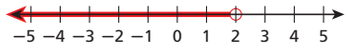
$-12 < -5c - 2$ — *Combine like terms. Since 2 is subtracted from −5c, add 2 to both sides to undo the subtraction.*

$+2 \quad +2$

$-10 < -5c$

$\frac{-10}{-5} > \frac{-5c}{-5}$ — *Since c is multiplied by −5, divide both sides by −5 to undo the multiplication. Change < to >.*

$2 > c$ (or $c < 2$)



B $-3(3 - x) < 4^2$

$-3(3 - x) < 4^2$ — *Distribute −3 on the left side.*

$-3(3) - (-3)x < 4^2$

$-9 + 3x < 4^2$

$-9 + 3x < 16$ — *Simplify the right side.*

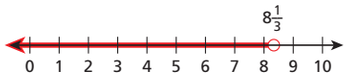
$-9 + 3x < 16$ — *Since −9 is added to 3x, add 9 to both sides to undo the addition.*

$+9 \quad +9$

$3x < 25$ — *Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.*

$\frac{3x}{3} < \frac{25}{3}$

$x < 8\frac{1}{3}$



C $\frac{4}{5}x + \frac{1}{2} > \frac{3}{5}$

$10\left(\frac{4}{5}x + \frac{1}{2}\right) > 10\left(\frac{3}{5}\right)$ — *Multiply both sides by 10, the LCD of the fractions.*

$10\left(\frac{4}{5}x\right) + 10\left(\frac{1}{2}\right) > 10\left(\frac{3}{5}\right)$ — *Distribute 10 on the left side.*

$8x + 5 > 6$ — *Since 5 is added to 8x, subtract 5 from both sides to undo the addition.*

$-5 \quad -5$

$8x > 1$

$\frac{8x}{8} > \frac{1}{8}$ — *Since x is multiplied by 8, divide both sides by 8 to undo the multiplication.*

$x > \frac{1}{8}$

CHECK IT OUT! Solve each inequality and graph the solutions.

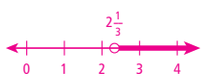
2a. $2m + 5 > 5^2$ $m > 10$

2b. $3 + 2(x + 4) > 3$ $x > -4$

2c. $\frac{5}{8} < \frac{3}{8}x - \frac{1}{4}$ $x > 2\frac{1}{3}$

2a.

2b.

2c.


COMMON ERROR ALERT

When the variable is on the right side of the inequality, some students may have difficulty rewriting it with the variable on the left. For example, a student might rewrite $2 > c$ as $c > 2$. Suggest that they choose a number for c that makes the inequality true. If $2 > 1$, then $1 < 2$, so if $2 > c$, then $c < 2$.

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Additional Examples

Example 1

Solve each inequality and graph the solutions.

A. $45 + 2b > 61$ $b > 8$

B. $8 - 3y \geq 29$ $y \leq -7$

Example 2

Solve each inequality and graph the solutions.

A. $2 - (-10) > -4t$ $t > -3$

B. $-4(2 - x) \leq 8$ $x \leq 4$

C. $\frac{2}{3}f + \frac{1}{2} > \frac{1}{3}$ $f > -\frac{1}{4}$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What operations would you use to solve the inequality? Why?

EXAMPLE 2

- What is the difference between simplifying and solving?

2 Teach

Guided Instruction

Review the properties of inequality and remind students that the inequality symbol must be reversed when multiplying or dividing both sides of an inequality by a negative number. Also remind students to check their solution by checking the endpoint and the inequality symbol.

Reaching All Learners *Through Visual Cues*

Give students several multi-step inequalities. Ask students to identify the operations performed on the variable (in order). Ask students to identify the steps to solve the inequality (in order). They can make a chart, as shown below.

Inequality	Operations on Variable	Operations to Solve
$5 > \frac{3p + 2}{4}$	1. Multiply by 3.	3. Divide by 3.
	2. Add 2.	2. Subtract 2.
	3. Divide by 4.	1. Multiply by 4.

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Additional Examples

Example 3

To rent a certain vehicle, Rent-A-Ride charges $55.00 per day with unlimited miles. The cost of renting a similar vehicle at We Got Wheels is $38.00 per day plus $0.20 per mile. For what number of miles is the cost at Rent-A-Ride less than the cost at We Got Wheels? more than 85 miles

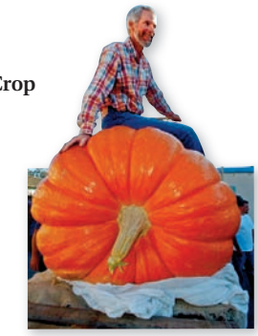
Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- How do you know which inequality symbol to use?

EXAMPLE 3 Gardening Application



To win the blue ribbon for the Heaviest Pumpkin Crop at the county fair, the average weight of John's two pumpkins must be greater than 819 lb. One of his pumpkins weighs 887 lb. What is the least number of pounds the second pumpkin could weigh in order for John to win the blue ribbon?

Let p represent the weight of the second pumpkin. The average weight of the pumpkins is the sum of each weight divided by 2.

(887 plus p) divided by 2 must be greater than 819.

$(887 + p) \div 2 > 819$

$$\frac{887 + p}{2} > 819$$

Since 887 + p is divided by 2, multiply both sides by 2 to undo the division.

$$2\left(\frac{887 + p}{2}\right) > 2(819)$$

$$887 + p > 1638$$

Since 887 is added to p, subtract 887 from both sides to undo the addition.

$$-887 \quad -887$$

$$p > 751$$

The second pumpkin must weigh more than 751 pounds.

Check Check the endpoint, 751.

$$\frac{887 + p}{2} = 819$$

$$\frac{887 + 751}{2} \mid 819$$

$$\frac{1638}{2} \mid 819$$

$$819 \mid 819 \checkmark$$

Check a number greater than 751.

$$\frac{887 + p}{2} > 819$$

$$\frac{887 + 755}{2} > 819$$

$$\frac{1642}{2} > 819$$

$$821 > 819 \checkmark$$

CHECK IT OUT!

3. The average of Jim's two test scores must be at least 90 to make an A in the class. Jim got a 95 on his first test. What scores can Jim get on his second test to make an A in the class?

Jim's score must be at least 85.

MATHEMATICAL PRACTICES

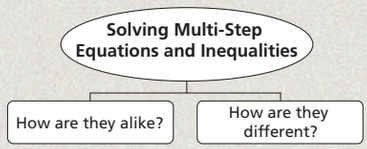
THINK AND DISCUSS

1. The inequality $v \geq 25$ states that 25 is the ___?___. (*value of v, minimum value of v,* or *maximum value of v*)

2. Describe two sets of steps for solving the inequality $\frac{x+5}{3} > 7$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer.



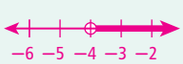
Paul Sakuma/AP/Wide World Photos

3 Close

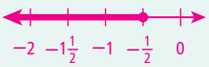
Summarize

Have students solve and graph the following inequalities.

$8 - 3x < 20$ $x > -4$

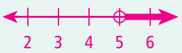


$5 - 13 \geq 16t$ $t \leq -\frac{1}{2}$



$-2(1 - y) < 4^2$ $y < 9$

$\frac{1}{5}p - \frac{1}{3} > \frac{2}{3}$ $p > 5$



ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. minimum value of v
2. 1. Multiply both sides by 3, and then subtract 5 from both sides.
   2. Rewrite the left side as $\frac{x}{3} + \frac{5}{3}$, subtract $\frac{5}{3}$ from both sides, and then multiply both sides by 3.
3. See Additional Answers.

2-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1

1. $2m + 1 > 13$
2. $2d + 21 \leq 11$
3. $6 \leq -2x + 2$
4. $4c - 7 > 5$
5. $\frac{4 + x}{3} > -4$
6. $1 < 0.2x - 0.7$
7. $\frac{3 - 2x}{3} \leq 7$
8. $2x + 5 \geq 2$

SEE EXAMPLE 2

9. $4(x + 2) > 6$
10. $\frac{1}{4}x + \frac{2}{3} < \frac{3}{4}$
11. $4 - x + 6^2 \geq 21$
12. $4 - x > 3(4 - 2)$
13. $0.2(x - 10) > -1.8$
14. $3(j + 41) \leq 35$

SEE EXAMPLE 3

15. **Business** A sales representative is given a choice of two paycheck plans. One choice includes a monthly base pay of $300 plus 10% commission on his sales. The second choice is a monthly salary of $1200. For what amount of sales would the representative make more money with the first plan? **sales of more than $9000**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–27	1
28–36	2
37	3

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each inequality and graph the solutions.

16. $4r - 9 > 7$
17. $3 \leq 5 - 2x$
18. $\frac{w + 3}{2} > 6$
19. $11w + 99 < 77$
20. $9 \geq \frac{1}{2}v + 3$
21. $-4x - 8 > 16$
22. $8 - \frac{2}{3}z \leq 2$
23. $f + 2\frac{1}{2} < -2$
24. $\frac{3n - 8}{5} \geq 2$
25. $-5 > -5 - 3w$
26. $10 > \frac{5 - 3p}{2}$
27. $2v + 1 > 2\frac{1}{3}$
28. $4(x + 3) > -24$
29. $4 > x - 3(x + 2)$
30. $-18 \geq 33 - 3h$
31. $-2 > 7x - 2(x - 4)$
32. $9 - (9)^2 > 10x - x$
33. $2a - (-3)^2 \geq 13$
34. $6 - \frac{x}{3} + 1 > \frac{2}{3}$
35. $12(x - 3) + 2x > 6$
36. $15 \geq 19 + 2(q - 18)$
37. **Communications** One cell phone company offers a plan that costs $29.99 and includes unlimited night and weekend minutes. Another company offers a plan that costs $19.99 and charges $0.35 per minute during nights and weekends. For what numbers of night and weekend minutes does the second company's plan cost more than the first company's plan? **starting at 29 min**

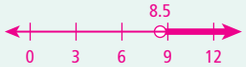
Solve each inequality and graph the solutions.

38. $-12 > -4x - 8$
39. $5x + 4 \leq 14$
40. $\frac{2}{3}x - 5 > 7$
41. $x - 3x > 2 - 10$
42. $5 - x - 2 > 3$
43. $3 < 2x - 5(x + 3)$
44. $\frac{1}{6} - \frac{2}{3}m \geq \frac{1}{4}$
45. $4 - (r - 2) > 3 - 5$
46. $0.3 - 0.5n + 1 \geq 0.4$
47. $6^2 > 4(x + 2)$
48. $-4 - 2n + 4n > 7 - 2^2$
49. $\frac{1}{4}(p - 10) \geq 6 - 4$
50. Use the inequality $-4t - 8 \leq 12$ to fill in the missing numbers.
    a. $t \geq$ **−5**
    b. $t + 4 \geq$ **−1**
    c. $t -$ **−5** ≥ 0
    d. $t + 10 \geq$ **5**
    e. $3t \geq$ **−15**
    f. $\frac{t}{\mathbf{1}} \geq -5$

Answers

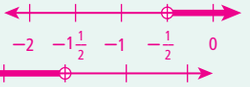
4. $c > 3$

5. $x > -16$

6. $x > 8.5$ 

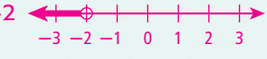
7. $x \geq -9$

8. $x \geq -\frac{3}{2}$

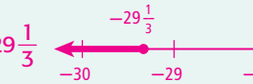
9. $x > -\frac{1}{2}$ 

10. $x < \frac{1}{3}$

11. $x \leq 19$

12. $x < -2$ 

13. $x > 1$

14. $j \leq -29\frac{1}{3}$ 

16–36, 38–49. See Additional Answers.

2-4 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 16–36, 38–49, 51–58
- **Average** 16–36, 38–49, 50–54, 68–70
- **Advanced** 16–36, 38–48 even, 50–54, 68–76

If you finished Examples **1–3**
- **Basic** 16–49, 51–62, 64–67, 77–86
- **Average** 16–37, 38–48 even, 50–70
- **Advanced** 16–26 even, 28–37, 38–48 even, 50–76

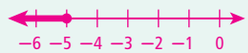
Homework Quick Check
Quickly check key concepts.
Exercises: 18, 32, 37, 46, 54, 60

Teaching Tip

Math Background There is more than one way to solve problems with fractions. **Exercise 44** could be solved by subtracting $\frac{1}{6}$ from both sides and then multiplying both sides by $-\frac{3}{2}$, by subtracting $\frac{1}{6}$ from both sides and then dividing both sides by $-\frac{2}{3}$, or by multiplying both sides by the LCD and then solving.

Answers

1. $m > 6$

2. $d \leq -5$ 

3. $x \leq -2$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 15, 37, 59–61

Construct viable arguments and critique the reasoning of others. Exercises 62–63

Learn It Online
State Resources Online

Teaching Tip **Reading Math** In **Exercise 53,** students may incorrectly interpret the inequality as $4n + 12 \leq 16$. Remind students that the product of a sum requires parentheses.

Teaching Tip **Inclusion** In **Exercise 60,** students must write an inequality using the formula for the area of a triangle where the height is a sum, $\frac{1}{2}(5)(2x + 3) < 55$. Students may distribute 5 and $\frac{1}{2}$ before using inverse operations, or they may multiply both sides by $\frac{2}{5}$ before using inverse operations.

MULTI-STEP TEST PREP **Exercise 61** involves using an inequality to find the number of CDs a band can have made. This exercise prepares students for the Multi-Step Test Prep.

Answers

51. $\frac{1}{2}x + 9 < 33$; $x < 48$

52. $6 \leq 4 - 2x$; $x \leq -1$

53. $4(x + 12) \leq 16$; $x \leq -8$

54. $\frac{1}{2}x + \frac{2}{3}x < 14$; $x < 12$

Write an inequality for each statement. Solve the inequality and graph the solutions.

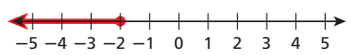
51. One-half of a number, increased by 9, is less than 33.

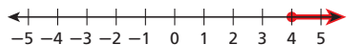
52. Six is less than or equal to the sum of 4 and $-2x$.

53. The product of 4 and the sum of a number and 12 is at most 16.

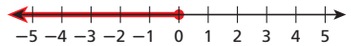
54. The sum of half a number and two-thirds of the number is less than 14.

Solve each inequality and match the solution to the correct graph.

55. $4x - 9 \geq 7$ — **A.**  — $x \geq 4$; B

56. $-6 \geq 3(x - 2)$ — **B.**  — $x \leq 0$; D

57. $-2x - 6 \geq -4 + 2$ — **C.** — $x \leq -2$; A

58. $\frac{1}{2} - \frac{1}{3}x \leq \left(\frac{2}{3} + \frac{1}{3}\right)^2$ — **D.**  — $x \geq -\frac{3}{2}$; C

59. Entertainment A digital video recorder (DVR) records television shows on an internal hard drive. To use a DVR, you need a subscription with a DVR service company. Two companies advertise their charges for a DVR machine and subscription service.

For what numbers of months will a consumer pay less for the machine and subscription at Easy Electronics than at Cable Solutions? **24 months or more**

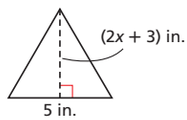
60. Geometry The area of the triangle shown is less than 55 square inches.

60a. $\frac{1}{2}(5)(2x + 3) < 55$

a. Write an inequality that can be used to find x.

b. Solve the inequality you wrote in part **a.** $x < 9.5$

c. What is the maximum height of the triangle? **22 in.**



MULTI-STEP TEST PREP

61. a. A band wants to create a CD of their last concert. They received a donation of $500 to cover the cost. The total cost is $350 plus $3 per CD. Complete the table to find a relationship between the number of CDs and the total cost.

Number	Process	Cost
1	350 + 3	353
2	350 + 3(2)	356
3	350 + 3(3)	359
10	350 + 3(10)	380
n	350 + 3(n)	350 + 3(n)

b. Write an equation for the cost c of the CDs based on the number of CDs n. $c = 350 + 3n$

c. Write an inequality that can be used to determine how many CDs can be made with the $500 donation. Solve the inequality and determine how many CDs the band can have made from the $500 donation.
$350 + 3n \leq 500$; $n \leq 50$; 50 CDs or fewer

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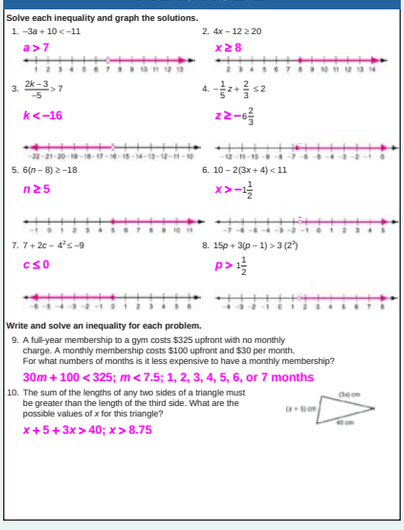
PRACTICE A

PRACTICE C

PRACTICE B



Solve each inequality and graph the solutions.

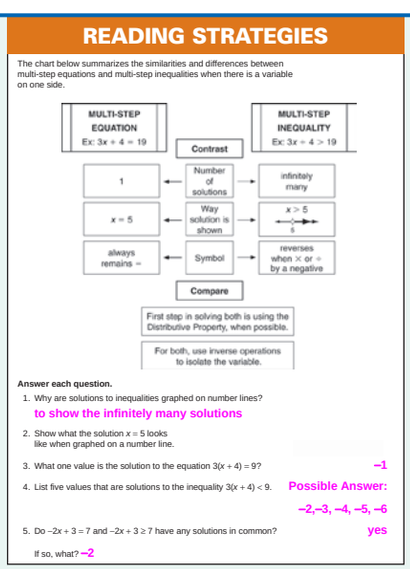
1. $-3a + 10 < -11$ — $a > 7$
2. $4x - 12 \geq 20$ — $x \geq 8$
3. $\frac{2k - 3}{-5} > 7$ — $k < -16$
4. $-\frac{1}{5}z + \frac{2}{3} \leq 2$ — $z \geq -6\frac{2}{3}$
5. $6(n - 8) \geq -18$ — $n \geq 5$
6. $10 - 2(3x + 4) < 11$ — $x > -1\frac{1}{2}$
7. $7 + 2c - 4^2 \leq -9$ — $c \leq 0$
8. $15p + 3(p - 1) > 3(2^2)$ — $p > 1\frac{1}{2}$

Write and solve an inequality for each problem.

9. A full-year membership to a gym costs $325 upfront with no monthly charge. A monthly membership costs $100 upfront and $30 per month. For what numbers of months is it less expensive to have a monthly membership?
$30m + 100 < 325$; $m < 7.5$; 1, 2, 3, 4, 5, 6, or 7 months

10. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side. What are the possible values of x for this triangle?
$x + 5 + 3x > 40$; $x > 8.75$

READING STRATEGIES



The chart below summarizes the similarities and differences between multi-step equations and multi-step inequalities when there is a variable on one side.

MULTI-STEP EQUATION Ex: $3x + 4 = 19$	Contrast	MULTI-STEP INEQUALITY Ex: $3x + 4 > 19$
1	Number of solutions	infinitely many
$x = 5$	Way solution is shown	$x > \frac{5}{5}$
always remains =	Symbol	reverses when × or ÷ by a negative

Compare

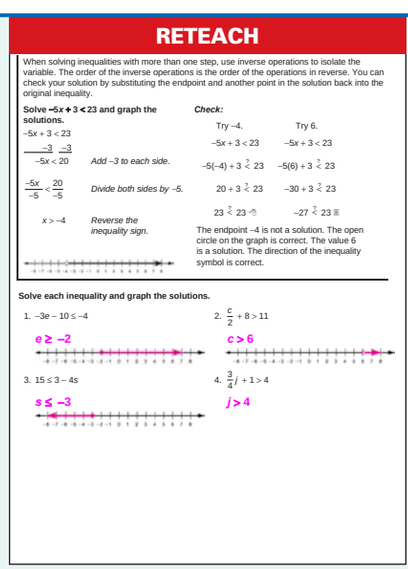
First step in solving both is using the Distributive Property, when possible.

For both, use inverse operations to isolate the variable.

Answer each question.

1. Why are solutions to inequalities graphed on number lines?
to show the infinitely many solutions
2. Show what the solution $x = 5$ looks like when graphed on a number line.
3. What one value is the solution to the equation $3(x + 4) = 9$? **–1**
4. List five values that are solutions to the inequality $3(x + 4) < 9$. Possible Answer: **–2, –3, –4, –5, –6**
5. Do $-2x + 3 = 7$ and $-2x + 3 \geq 7$ have any solutions in common? **yes**
If so, what? **–2**

RETEACH



When solving inequalities with more than one step, use inverse operations to isolate the variable. The order of the inverse operations is the order of the operations in reverse. You can check your solution by substituting the endpoint and another point in the solution back into the original inequality.

Solve $-5x + 3 < 23$ and graph the solutions.

$-5x + 3 < 23$	
$\frac{-3}{-5x} < \frac{-3}{20}$	Add –3 to each side.
$\frac{-5x}{-5} < \frac{20}{-5}$	Divide both sides by –5.
$x > -4$	Reverse the inequality sign.

Check:

Try –4. $-5x + 3 < 23$; $-5(-4) + 3 \stackrel{?}{<} 23$; $20 + 3 \stackrel{?}{<} 23$; $23 \stackrel{?}{<} 23$ ✗

Try 6. $-5x + 3 < 23$; $-5(6) + 3 \stackrel{?}{<} 23$; $-30 + 3 \stackrel{?}{<} 23$; $-27 \stackrel{?}{<} 23$ ✓

The endpoint –4 is not a solution. The open circle on the graph is correct. The value 6 is a solution. The direction of the inequality symbol is correct.

Solve each inequality and graph the solutions.

1. $-3e - 10 \leq -4$ — $e \geq -2$
2. $\frac{c}{2} + 8 > 11$ — $c > 6$
3. $15 \leq 3 - 4s$ — $s \leq -3$
4. $\frac{3}{4}j + 1 > 4$ — $j > 4$

62. Critical Thinking What is the least whole number that is a solution of $4r - 4.9 > 14.95$? $r = 5$

63. Write About It Describe two sets of steps to solve $2(x + 3) > 10$.

1. Divide both sides by 2, and then subtract 3 from both sides.
2. Distribute 2 on the left side, subtract 6 from both sides, and then divide both sides by 2.

TEST PREP

64. What are the solutions of $3y > 2x + 4$ when $y = 6$?

(A) $7 > x$ (B) $x > 7$ (C) $x > 11$ (D) $11 > x$

65. Cecilia has $30 to spend at a carnival. Admission costs $5.00, lunch will cost $6.00, and each ride ticket costs $1.25. Which inequality represents the number of ride tickets x that Cecilia can buy?

(F) $30 - (5 - 6) + 1.25x \leq 30$ (H) $30 - (5 + 6) \leq 1.25x$

(G) $5 + 6 + 1.25x \leq 30$ (J) $30 + 1.25x \leq 5 + 6$

66. Which statement is modeled by $2p + 5 < 11$?

(A) The sum of 5 and 2 times p is at least 11.

(B) Five added to the product of 2 and p is less than 11.

(C) Two times p plus 5 is at most 11.

(D) The product of 2 and p added to 5 is 11.

67. Gridded Response A basketball team scored 8 points more in its second game than in its first. In its third game, the team scored 42 points. The total number of points scored in the three games was more than 150. What is the least number of points the team might have scored in its *second* game? 59

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

68. $3(x + 2) - 6x + 6 \leq 0$ $x \geq 4$

69. $-18 > -(2x + 9) - 4 + x$ $x > 5$

70. $\frac{2 + x}{2} - (x - 1) > 1$ $x < 2$

Write an inequality for each statement. Graph the solutions.

71. x is a positive number. $x > 0$

72. x is a negative number. $x < 0$

73. x is a nonnegative number. $x \geq 0$

74. x is not a positive number. $x \leq 0$

75. x times negative 3 is positive. $-3x > 0$

76. The opposite of x is greater than 2. $-x > 2$

TEST PREP DOCTOR

For **Exercise 66,** the verbal translation for the left side of the inequality is correct for all four choices. Choice **D** can be eliminated because *is* means "equal." "At least" and "at most" both indicate or-equal-to symbols.

Answers

68–76. For graphs, see Additional Answers.

Journal

Write about the differences and similarities in solving a multi-step inequality and a multi-step equation. Include a comparison of the solutions and the methods of checking the solutions.

ALTERNATIVE ASSESSMENT

Instruct students to write a real-world problem that can be modeled by an inequality that requires at least two steps to solve. Ask students to solve the inequality explaining each step. Students should explain the meaning of the solution in the context of the problem.

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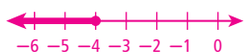
2-4 Lesson Quiz

Solve each inequality and graph the solutions.

1. $13 - 2x \geq 21$ $x \leq -4$



2. $-11 + 2 < 3p$ $p > -3$

3. $2^3 < -2(3 - t)$ $t > 7$

4. $\frac{1}{3}n + \frac{1}{2} < \frac{2}{3}$ $n < \frac{1}{2}$

5. A video store has two movie-rental plans. Plan A includes a $25 membership fee plus $1.25 for each movie rental. Plan B costs $40 for unlimited movie rentals. For what number of movie rentals is plan B less than plan A?
more than 12 movies

Also available online

PROBLEM SOLVING

Write and solve an inequality for each situation.

1. Jillene is playing in a basketball tournament and scored 24 points in her first game. If she averages over 20 points for both games, she will receive a trophy. How many points can Jillene score in the second game and receive a trophy?
$\frac{p + 24}{2} > 20$; $p > 16$

2. Marcus has accepted a job selling cell phones. He will be paid $1500 plus 15% of his sales each month. He needs to earn at least $2430 to pay his bills. For what amount of sales will Marcus be able to pay his bills?
$1500 + 0.15s \geq 2430$; $s \geq 6200$

3. A 15-foot-tall cedar tree is growing at a rate of 2 feet per year beneath power lines that are 58 feet above the ground. The power company will have to prune or remove the tree before it reaches the lines. How many years can the power company wait before taking action?
$15 + 2y < 58$; $y < 21.5$

4. Binh brought $23 with her to the county fair. She purchased a $5 T-shirt and now wants to buy some locally grown plants for $2.50 each. What are the numbers of plants that she can purchase with her remaining money?
$5 + 2.5p \leq 23$; $p \leq 7.2$; 0, 1, 2, 3, 4, 5, 6, or 7 plants

Benedict, Ricardo, and Charlie are considering opportunities for summer work. The table below shows the jobs open to them and the pay for each. Use this information to answer questions 5–7.

Job	Pay
Mowing Lawns	$15 per lawn
Baby-Sitting	$5.50 per hour
Tutoring	$9 per session

5. Benedict has saved $91 from last year and would like to baby-sit to earn enough to buy a mountain bike. A good quality bike costs at least $300. What numbers of hours h can Benedict baby-sit to reach his goal?
A $h \geq 14$ C $h \geq 38$
B $h \geq 23$ D $h \geq 71$

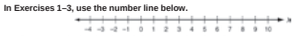
6. Ricardo has agreed to tutor for the school. He owes his older brother $59 and would like to end the summer with at least $400 in savings. How many sessions s can Ricardo tutor to meet his goal?
F $s \geq 31$ H $s \geq 51$
G $s \geq 38$ J $s \geq 83$

7. Charlie has agreed to mow his neighbor's lawn each week and will also baby-sit some hours. If he makes $100 or more each week, his parents will charge him rent. How many hours h should Charlie agree to baby-sit each week to avoid paying rent?
A $h \leq 15$ C $h \leq 21$
B $h \geq 15$ D $h \geq 21$

CHALLENGE

As you have already seen, when you solve a linear equation, you find a definite value. If the variable is equal to 3, then the number line is divided into three parts: the number 3, those numbers greater than 3, and those numbers less than 3. When you deal with multiple inequalities, you divide the number line into even more parts.

In Exercises 1–3, use the number line below.



1. Draw a graph showing the solutions to $x \geq 3$ and $x < 7$.

2. What integers satisfy both of these inequalities? 3, 4, 5, 6

3. What inequalities describe your response in Exercise 2?
$x \geq 3$ and $x \leq 6$ where x is an integer.

On a certain freeway, the minimum allowable speed is 40 miles per hour, and the maximum allowable speed is 55 miles per hour.

Use the information above to answer Exercises 4–6.

4. Write an inequality for each driving situation.
a. A motorist is driving too fast. $x > 55$
b. A motorist is driving too slow. $x > 0$ and $x < 40$

5. Is it technically correct to say that the speed limit is exactly 55 miles per hour?
No, since there is a lower limit as well as an upper limit

6. Write multiple inequalities for each situation.
a. The driver's speed is within the speed limits. $x \geq 40$ and $x \leq 55$
b. The driver's speed is outside of the speed limits. $x > 0$ and $x < 40$ or $x > 55$

People use multiple inequalities when they deal with real-estate situations.

7. John and Marsha would like to buy a house priced at $185,000, plus or minus $5,000.
a. What are the maximum and minimum house price that they are willing to pay? maximum: $190,000; minimum: $180,000
b. Write a pair of inequalities describing their price range.
$x \geq 180{,}000$ and $x \leq 190{,}000$

8. Use inequalities to describe the following sentence: "We sell houses priced at $80,000, plus or minus $5,000, to houses priced at $250,000, plus or minus $5,000."
$x \geq 75{,}000$ and $x \leq 255{,}000$

2-5 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve inequalities that contain variable terms on both sides.

Algebra Lab
In *Algebra Lab Activities*

Technology Lab
In *Technology Lab Activities*

Online Edition
Tutorial Videos

Countdown

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Warm Up

Solve each equation.

1. $2x = 7x + 15$ $x = -3$
2. $3y - 21 = 4 - 2y$ $y = 5$
3. $2(3z + 1) = -2(z + 3)$ $z = -1$
4. $3(p - 1) = 3p + 2$ no solution
5. Solve and graph $5(2 - b) > 5^2$. $b < -3$

Also available online

Math Humor

Q: What did Miss Manners say to the inequality symbol?

A: It's not polite to point.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

Learn It Online
State Resources Online

COMMON CORE

2-5 Solving Inequalities with Variables on Both Sides

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Also* **CC.9-12.A.CED.1***

Objective
Solve inequalities that contain variable terms on both sides.

Who uses this?
Business owners can use inequalities to find the most cost-efficient services. (See Example 2.)

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities like you solved equations with variables on both sides.

Use the properties of inequality to "collect" all the variable terms on one side and all the constant terms on the other side.

EXAMPLE 1 **Solving Inequalities with Variables on Both Sides**

Solve each inequality and graph the solutions.

A $x < 3x + 8$

$x < 3x + 8$ — *To collect the variable terms on one side, subtract x from both sides.*
$-x \quad -x$
$0 < 2x + 8$ — *Since 8 is added to 2x, subtract 8 from both sides to undo the addition.*
$-8 \quad -8$
$-8 < 2x$
$\frac{-8}{2} < \frac{2x}{2}$ — *Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.*
$-4 < x$ (or $x > -4$)

Helpful Hint
Your first step can also be to subtract $3x$ from both sides to get $-2x < 8$. When you divide by a negative number, remember to reverse the inequality symbol.

B $6x - 1 \leq 3.5x + 4$

$6x - 1 \leq 3.5x + 4$ — *Subtract 6x from both sides.*
$-6x \quad -6x$
$-1 \leq -2.5x + 4$ — *Since 4 is added to −2.5x, subtract 4 from both sides to undo the addition.*
$-4 \quad -4$
$-5 \leq -2.5x$
$\frac{-5}{-2.5} \geq \frac{-2.5x}{-2.5}$ — *Since x is multiplied by −2.5, divide both sides by−2.5 to undo the multiplication. Reverse the inequality symbol.*
$2 \geq x$

CHECK IT OUT! **Solve each inequality and graph the solutions.**

1a. $4x \geq 7x + 6$ $x \leq -2$

1b. $5t + 1 < -2t - 6$ $t < -1$

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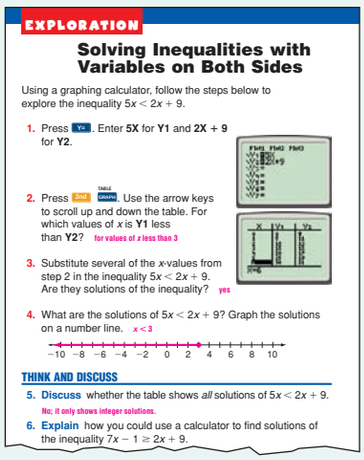
1 Introduce



EXPLORATION

Solving Inequalities with Variables on Both Sides

Using a graphing calculator, follow the steps below to explore the inequality $5x < 2x + 9$.

1. Press Y=. Enter **5X** for **Y1** and **2X + 9** for **Y2**.
2. Press 2nd TABLE GRAPH. Use the arrow keys to scroll up and down the table. For which values of *x* is **Y1** less than **Y2**? for values of *x* less than 3
3. Substitute several of the *x*-values from step 2 in the inequality $5x < 2x + 9$. Are they solutions of the inequality? yes
4. What are the solutions of $5x < 2x + 9$? Graph the solutions on a number line. $x < 3$

THINK AND DISCUSS

5. **Discuss** whether the table shows *all* solutions of $5x < 2x + 9$. No; it only shows integer solutions.
6. **Explain** how you could use a calculator to find solutions of the inequality $7x - 1 \geq 2x + 9$.

Motivate

Present two ski-ticket plans to students. Plan A costs \$45 per day and includes equipment and the lift ticket; plan B costs \$350 for the season and includes equipment but not the \$20 lift ticket. Discuss with students how to determine the number of days for which plan A costs less than plan B. $45x < 350 + 20x$ Tell students that they could use an inequality with variables on both sides, which they will learn to solve in this lesson.

Explorations and answers are provided in your online resources.

EXAMPLE 2 ***Business Application***

The *Daily Info* charges a fee of $650 plus $80 per week to run an ad. The *People's Paper* charges $145 per week. For how many weeks will the total cost at *Daily Info* be less expensive than the cost at *People's Paper*?

Let w be the number of weeks the ad runs in the paper.

Daily Info fee	plus	$80 per week	times	number of weeks	is less expensive than	*People's Paper* charge per week	times	number of weeks.
$650	+	$80	•	w	<	$145	•	w

$$650 + 80w < 145w$$

$$-80w \quad -80w$$

Subtract 80w from both sides.

$$650 < 65w$$

Since w is multiplied by 65, divide both sides by 65 to undo the multiplication.

$$\frac{650}{65} < \frac{65w}{65}$$

$$10 < w$$

The total cost at *Daily Info* is less than the cost at *People's Paper* if the ad runs for more than 10 weeks.

CHECK IT OUT! **2.** A-Plus Advertising charges a fee of $24 plus $0.10 per flyer to print and deliver flyers. Print and More charges $0.25 per flyer. For how many flyers is the cost at A-Plus Advertising less than the cost at Print and More? **more than 160 flyers**

You may need to simplify one or both sides of an inequality before solving it. Look for like terms to combine and places to use Distributive Property.

EXAMPLE 3 **Simplifying Each Side Before Solving**

Solve each inequality and graph the solutions.

A $6(1 - x) < 3x$

$$6(1 - x) < 3x$$

Distribute 6 on the left side of the inequality.

$$6(1) - 6(x) < 3x$$

$$6 - 6x < 3x$$

Add 6x to both sides so that the coefficient of x is positive.

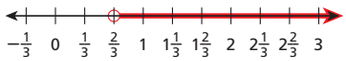
$$+6x \quad +6x$$

$$6 < 9x$$

Since x is multiplied by 9, divide both sides by 9 to undo the multiplication.

$$\frac{6}{9} < \frac{9x}{9}$$

$$\frac{2}{3} < x$$



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Additional Examples

Example 1

Solve each inequality and graph the solutions.

A. $y \leq 4y + 18$ $y \geq -6$

B. $4m - 3 < 2m + 6$ $m < \frac{9}{2}$

Example 2

The Home Cleaning Company charges $312 to power-wash the siding of a house plus $12 for each window. Power Clean charges $36 per window, and the price includes power-washing the siding. How many windows must a house have to make the total cost from The Home Cleaning Company less expensive than Power Clean?

more than 13 windows

Example 3

Solve each inequality and graph the solutions.

A. $2(k - 3) > 6 + 3k - 3$

$k < -9$

B. $0.9y \geq 0.4y - 0.5$ $y \geq -1$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- How do you decide on which side of the inequality to gather the variable terms?

EXAMPLE 2

- How do you know which inequality symbol to use?

EXAMPLE 3

- How can you tell when an expression is simplified?

2 Teach

Guided Instruction

Before solving inequalities with variables on both sides, review the steps used to solve equations with variables on both sides.

Teaching Tip **Visual Cues** To help students focus on simplifying first, tell them to cover up the right side of the inequality and simplify the left side. Then cover the left side and simplify the right side.

Reaching All Learners

Through Communication

Give students one inequality at a time. For each, ask the following questions.

- On which side of the inequality would you gather the variable terms? Why?
- Would the inequality symbol need to be reversed?

Example: $3x + 1 < 4x$

Possible answers: right, to keep the variable positive; not unless you rewrite the solutions of the inequality with the variable on the left

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Additional Examples

Example 4

Solve each inequality.

A. $2x - 7 \leq 5 + 2x$
all real numbers

B. $2(3y - 2) - 4 \geq 3(2y + 7)$
no solutions

Also available online

Mathematical Practices Questioning Strategies

EXAMPLE 4

- How can you tell whether the solution of an inequality is *all real numbers* or whether the inequality has *no solutions*?

Helpful Hint

In Example 3B, you can also multiply each term in the inequality by the same power of 10 to clear the decimals.
$10(1.6x) \leq 10(-0.2x) + 10(0.9)$
$16x \leq -2x + 9$

Solve each inequality and graph the solutions.

B $1.6x \leq -0.2x + 0.9$

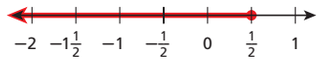
$1.6x \leq -0.2x + 0.9$
$+0.2x \quad +0.2x$
$1.8x \leq 0.9$

Since −0.2x is added to 0.9, subtract −0.2x from both sides. Subtracting −0.2x is the same as adding 0.2x.

$\frac{1.8x}{1.8} \leq \frac{0.9}{1.8}$

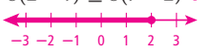
Since x is multiplied by 1.8, divide both sides by 1.8 to undo the multiplication.

$x \leq \frac{1}{2}$



CHECK IT OUT! **Solve each inequality and graph the solutions. Check your answer.**

3a. $5(2 - r) \geq 3(r - 2)$ $r \leq 2$



3b. $0.5x - 0.3 + 1.9x < 0.3x + 6$ $x < 3$

Some inequalities are true no matter what value is substituted for the variable. For these inequalities, all real numbers are solutions.

Some inequalities are false no matter what value is substituted for the variable. These inequalities have no solutions.

If both sides of an inequality are fully simplified and the same variable term appears on both sides, then the inequality has all real numbers as solutions or it has no solutions. Look at the other terms in the inequality to decide which is the case.

EXAMPLE 4 All Real Numbers as Solutions or No Solutions

Solve each inequality.

A $x + 5 \geq x + 3$

$x + 5 \geq x + 3$

The same variable term (x) appears on both sides. Look at the other terms.

For any number x, adding 5 will always result in a greater number than adding 3.

All values of x make the inequality true.
All real numbers are solutions.

B $2(x + 3) < 5 + 2x$

$2x + 6 < 5 + 2x$ *Distribute 2 on the left side.*

The same variable term ($2x$) appears on both sides. Look at the other terms.

For any number $2x$, adding 6 will never result in a lesser number than adding 5.

No values of x make the inequality true.
There are no solutions.

CHECK IT OUT! **Solve each inequality.**

4a. $4(y - 1) \geq 4y + 2$
no solutions

4b. $x - 2 < x + 1$
all real numbers

3 Close

Summarize

Have students solve the following and explain each step.

$a + 8 > 3a$ $a < 4$

$2(x - 3) > x - 5^2$ $x > -19$

$4(1 - t) < -2(2t + 3)$ no solutions

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

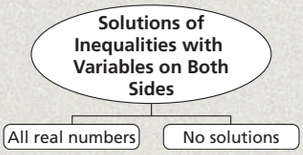
***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

1. Explain how you would collect the variable terms to solve the inequality $5c - 4 > 8c + 2$.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of an inequality of the type indicated.



2-5 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Solve each inequality and graph the solutions.

1. $2x > 4x - 6$ $x < 3$
2. $7y + 1 \le y - 5$ $y \le -1$
3. $27x + 33 > 58x - 29$ $x < 2$
4. $-3r < 10 - r$ $r > -5$
5. $5c - 4 > 8c + 2$ $c < -2$
6. $4.5x - 3.8 \ge 1.5x - 2.3$ $x \ge \frac{1}{2}$

SEE EXAMPLE 2

7. **School** The school band will sell pizzas to raise money for new uniforms. The supplier charges \$100 plus \$4 per pizza. If the band members sell the pizzas for \$7 each, how many pizzas will they have to sell to make a profit? at least 34 pizzas

SEE EXAMPLE 3

Solve each inequality and graph the solutions.

8. $5(4 + x) \le 3(2 + x)$ $x \le -7$
9. $-4(3 - p) > 5(p + 1)$ $p < -17$
10. $2(6 - x) < 4x$ $x > 2$
11. $4x > 3(7 - x)$ $x > 3$
12. $\frac{1}{2}f + \frac{3}{4} \ge \frac{1}{4}f$ $f \ge -3$
13. $-36.72 + 5.65t < 0.25t$ $t < 6.8$

SEE EXAMPLE 4

Solve each inequality.

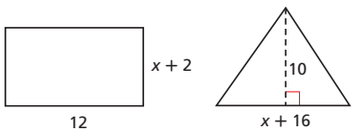
14. $2(x - 2) \le -2(1 - x)$ all real numbers
15. $4(y + 1) < 4y + 2$ no solutions
16. $4v + 1 < 4v - 7$ no solutions
17. $b - 4 \ge b - 6$ all real numbers
18. $3(x - 5) > 3x$ no solutions
19. $2k + 7 \ge 2(k + 14)$ no solutions

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

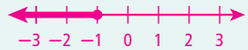
20. $3x \le 5x + 8$ $x \ge -4$
21. $9y + 3 > 4y - 7$ $y > -2$
22. $1.5x - 1.2 < 3.1x - 2.8$ $x > 1$
23. $7 + 4b \ge 3b$ $b \ge -7$
24. $7 - 5t < 4t - 2$ $t > 1$
25. $2.8m - 5.2 > 0.8m + 4.8$ $m > 5$

26. **Geometry** For what values of x is the area of the rectangle greater than the area of the triangle? $x > 8$



Answers

1.

2. 

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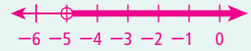
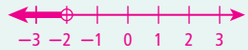
4. 

5. 

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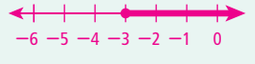
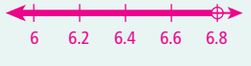
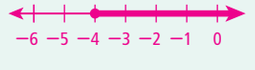
12. 

13. 

20. 

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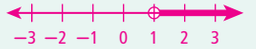
22. 

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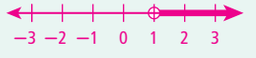
24. 

25.

Answers to *Think and Discuss*

1. Possible answer: Subtract $5c$ from both sides of the inequality so that all variable terms are on the right side. Then subtract 2 from both sides so that all constant terms are on the left side.
2. See Additional Answers.

2-5 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**

Basic 20–26, 39–42, 44, 49, 52–56, 60
Average 20–26, 42, 44, 49, 52–56, 66
Advanced 20–26, 42, 44, 49, 52–56, 66

If you finished Examples **1–4**

Basic 20–49, 51–58, 60–65
Average 20–38, 40–48 even, 49–51, 56–66
Advanced 24, 26, 27–38, 40–48 even, 49–51, 56–59, 61–72

Homework Quick Check
Quickly check key concepts.
Exercises: 24, 26, 28, 38, 40, 44

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 7, 26, 49–51, 56–57

Reason abstractly and quantitatively. Exercises 70–71

Construct viable arguments and critique the reasoning of others. Exercises 58–60, 72

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 51** requires writing and solving multi-step inequalities in order to solve problems relating cost and profit. This exercise prepares students for the Multi-Step Test Prep.

Answers

27. $-3\ -2\ -1\ 0\ 1\ 2\ 3$

28. $0\ 2\ 4\ 6\ 8$

29. $0\ 2\ 4\ 6\ 8$

30. $0\ 1\ 2\ 3\ 4\ 5\ 6$

31. $-6\ -5\ -4\ -3\ -2\ -1\ 0$

32. $0\ 1\ 2\ 3\ 4\ 5\ 6$

39. $-9\ -8\ -7\ -6\ -5$

40. $-6\ -5\ -4\ -3\ -2\ -1\ 0$

41. $0\ 1\ 2\ 3\ 4\ 5\ 6$

42. $16\ \ 16\frac{1}{3}\ \ 16\frac{2}{3}\ \ 17$

43. $-3\ -2\ -1\ 0\ 1\ 2\ 3$

44. $0\ 1\ 2\ 3\ 4\ 5\ 6$

45. $-3\ -2\ -1\ 0\ 1\ 2\ 3$

46. $-6\ -5\ -4\ -3\ -2\ -1\ 0$

47. $-8\ \ -6\ \ -4\ \ -2\ \ 0$

48. $20\ \ \frac{7}{3}\ \ \frac{8}{3}\ \ 3\ \ \frac{10}{3}\ \ \frac{11}{3}\ \ 4$

50e. $11\ 12\ 13\ 14\ 15\ 16$

PRACTICE A

PRACTICE C

PRACTICE B

Solve each inequality and graph the solutions.

1. $2x + 30 \geq 7x$ — $x \leq 6$
2. $2k + 6 < 5k - 3$ — $k > 3$
3. $3b - 2 \leq 2b + 1$ — $b \leq 3$
4. $2(3n + 7) > 5n$ — $n > -14$
5. $5s - 9 < 2(s - 6)$ — $s < -1$
6. $-3(3x + 5) \geq -5(2x - 2)$ — $x \geq 25$
7. $1.4z + 2.2 > 2.6z - 0.2$ — $z < 2$
8. $\frac{7}{8}p - \frac{1}{4} \leq \frac{1}{2}p$ — $p \leq \frac{2}{3}$

Solve each inequality.

9. $v + 1 > v - 6$ — all real numbers
10. $3(x + 4) \leq 3x$ — no solutions
11. $-2(8 - 3x) \geq 6x + 2$ — no solution

Write and solve an inequality for each problem.

12. Ian wants to promote his band on the Internet. Site A offers website hosting for $4.95 per month with a $49.95 startup fee. Site B offers website hosting for $9.95 per month with no startup fee. For how many months would Ian need to keep the website for Site B to be less expensive than Site A?
    $9.95m < 4.95m + 49.95$; $m < 9.99$; for 0 to 9 months
13. For what values of x is the area of the rectangle greater than the perimeter?
    $7(x + 2) > 7 + (x + 2) + 7 + (x + 2)$; $x > 0.8$

Independent Practice

For Exercises	See Example
20–25	1
26	2
27–32	3
33–38	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each inequality and graph the solutions.

27. $4(2 - x) \leq 5(x - 2)$ $x \geq 2$
28. $-3(n + 4) < 6(1 - n)$ $n < 6$
29. $9(w + 2) \leq 12w$ $w \geq 6$
30. $4.5 + 1.3t > 3.8t - 3$ $t < 3$
31. $\frac{1}{2}r + \frac{2}{3} \geq \frac{1}{3}r$ $r \geq -4$
32. $2(4 - n) < 3n - 7$ $n > 3$
33. no solutions

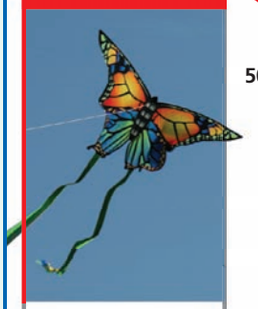
Solve each inequality.

33. $3(2 - x) < -3(x - 1)$ — all real numbers
34. $7 - y > 5 - y$ — all real numbers
35. $3(10 + z) \leq 3z + 36$ — all real numbers
36. $-5(k - 1) \geq 5(2 - k)$ — no solutions
37. $4(x - 1) \leq 4x$ — all real numbers
38. $3(v - 9) \geq 15 + 3v$ — no solutions

Solve each inequality and graph the solutions.

39. $3t - 12 > 5t + 2$ $t < -7$
40. $-5(y + 3) - 6 < y + 3$ $y > -4$
41. $3x + 9 - 5x < x$ $x > 3$
42. $18 + 9p > 12p - 31$ $p < 16\frac{1}{3}$
43. $2(x - 5) < -3x$ $x < 2$
44. $-\frac{2}{5}x \leq \frac{4}{5} - \frac{3}{5}x$ $x \leq 4$
45. $-2(x - 7) - 4 - x < 8x + 32$ $x > -2$
46. $-3(2r - 4) \geq 2(5 - 3r)$ all real numbers
47. $-7x - 10 + 5x \geq 3(x + 4) + 8$ $x \leq -6$
48. $-\frac{1}{3}(n + 8) + \frac{1}{3}n \leq 1 - n$ $n \leq \frac{11}{3}$

LINK Recreation



The American Kitefliers Association has over 4000 members in 35 countries. Kitefliers participate in festivals, competitions, and kite-making workshops.

49. **Recreation** A red kite is 100 feet off the ground and is rising at 8 feet per second. A blue kite is 180 feet off the ground and is rising at 5 feet per second. How long will it take for the red kite to be higher than the blue kite? Round your answer to the nearest second. 27 s

50. **Education** The table shows the enrollment in Howard High School and Phillips High School for three school years.

School Enrollment			
	Year 1	Year 2	Year 3
Howard High School	1192	1188	1184
Phillips High School	921	941	961

a. How much did the enrollment change each year at Howard? decreased by 4

b. Use the enrollment in year 1 and your answer from part **a** to write an expression for the enrollment at Howard in any year x. $1192 - 4(x - 1)$

c. How much did the enrollment change each year at Phillips? increased by 20

d. Use the enrollment in year 1 and your answer from part **c** to write an expression for the enrollment at Phillips in any year x. $921 + 20(x - 1)$

e. Assume that the pattern in the table continues. Use your expressions from parts **b** and **d** to write an inequality that can be solved to find the year in which the enrollment at Phillips High School will be greater than the enrollment at Howard High School. Solve your inequality and graph the solutions.
$1192 - 4(x - 1) < 921 + 20(x - 1)$; $x \geq 13$ where x is a whole number

MULTI-STEP TEST PREP

51. a. The school orchestra is creating a CD of their last concert. The total cost is $400 + 4.50 per CD. Write an expression for the cost of creating the CDs based on the number of CDs n. $400 + 4.50n$

b. The orchestra plans to sell the CDs for $12. Write an expression for the amount the orchestra earns from the sale of n CDs. $12n$

c. In order for the orchestra to make a profit, the amount they make selling the CDs must be greater than the cost of creating the CDs. Write an inequality that can be solved to find the number of CDs the orchestra must sell in order to make a profit. Solve your inequality. $400 + 4.50n < 12n$; $n > 53\frac{1}{3}$; 54 CDs or more

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READING STRATEGIES

When solving an inequality with variables on both sides, you have a choice: Collect the variables on the left, or collect the variables on the right. There are advantages to each, depending on the inequality. Either way, you should get the same answer.

$x - 7 > 4x + 2$

$-4x \quad -4x$

$-3x - 7 > 2$

$+7 \quad +7$

$\frac{-3x}{-3} > \frac{9}{-3}$

$x < -3$

Collect variables on the left.

Analysis

With this choice, we divide by a negative. So we must remember to change the direction of the inequality symbol.

$x - 7 > 4x + 2$

$-7 > 3x + 2$

$-2 \quad -2$

$\frac{-9}{3} > \frac{3x}{3}$

$-3 > x$

$x < -3$

Collect variables on the right.

Analysis

With this choice, the solution is "backwards". We must flip the solutions so that we can read it from left to right.

Answer each question.

1. Kirby often forgets to switch the inequality symbol when dividing by a negative. When solving $3x - 10 \geq x$, should Kirby collect the variables on the left or the right? Why?
   Left; so he divides by a positive number.
2. Rewrite the inequality $8 < t$ so that the variable is on the left. $t > 8$
3. Solve the inequality $-2(n + 3) > -4n + 8$ by collecting variables on the left, then by collecting variables on the right. Which seems better in this case? Why?
   $n > 7$; left; you do not have to switch the inequality symbol.

RETEACH

Variables must be collected on the same side of an inequality before the inequality can be solved. If you collect the variables so that the variable term is positive, you will not have to multiply or divide by a negative number.

Solve $x > 8(x - 7)$. Collect the variables on the left.

$x > 8(x - 7)$

$x > 8x - 56$ Distribute.

$\frac{-8x}{-7x} > \frac{-8x}{-56}$ Add $-8x$ to both sides.

$\frac{-7x}{-7} > \frac{-56}{-7}$ Divide both sides by -7.

$x < 8$ Reverse the sign.

Solve $x > 8(x - 7)$. Collect the variables on the right.

$x > 8(x - 7)$

$x > 8x - 56$ Distribute.

$\frac{-x}{0} > \frac{-x}{7x - 56}$ Add $-x$ to both sides.

$\frac{+56}{56} > \frac{+56}{7x}$

$\frac{56}{7} > \frac{7x}{7}$ Divide both sides by 7.

$8 > x$

$x < 8$

Notice that if you want to have the variable on the left to make graphing solutions easier, you may still need to switch the inequality sign, even if you did not multiply or divide by a negative number.

Write the first step you would take to solve each inequality if you wanted to keep the variable positive.

1. $6y < 10y + 1$ — add $-6y$ to both sides
2. $4p - 2 \geq 3p$ — add $-3p$ to both sides
3. $5 - 3r \leq 6r$ — add $3r$ to both sides

Solve each inequality.

4. $8c + 4 > 4(c - 3)$ — $c > -4$
5. $5(x - 1) < 3x + 10 - 8x$ — $x < \frac{3}{2}$
6. $-8 + 4a - 12 > 2a + 10$ — $a > 15$

52. $2x + 4 > \frac{2}{3}x$; $x > -3$

Write an inequality to represent each relationship. Solve your inequality.

52. Four more than twice a number is greater than two-thirds of the number.

53. Ten less than five times a number is less than six times the number decreased by eight. $5x - 10 < 6x - 8$; $x > -2$

54. The sum of a number and twenty is less than four times the number decreased by one. $x + 20 < 4x - 1$; $x > 7$

55. $\frac{3}{4}x \geq x - 5$; $x \leq 20$

55. Three-fourths of a number is greater than or equal to five less than the number.

56. **Entertainment** Use the table to determine how many movies you would have to rent for Video View to be less expensive than Movie Place. **20 movies or more**

	Membership Fee ($)	Cost per Rental ($)
Movie Place	None	2.99
Video View	19.99	1.99

57. **Geometry** In an acute triangle, all angles measure less than 90°. Also, the sum of the measures of any two angles is greater than the measure of the third angle. Can the measures of an acute triangle be x, $x - 1$, and $2x$? Explain.

58. **Write About It** Compare the steps you would follow to solve an inequality to the steps you would follow to solve an equation.

59. **Critical Thinking** How can you tell just by looking at the inequality $x > x + 1$ that it has no solutions? ***x* can never be greater than itself plus 1.**

60. **ERROR ANALYSIS** Two students solved the inequality $5x < 3 - 4x$. Which is incorrect? Explain the error.

B is incorrect; the student should have added $4x$ to both sides to undo the subtraction.

A

$5x < 3 - 4x$

$+4x \quad +4x$

$9x < 3$

$x < \frac{1}{3}$

B

$5x < 3 - 4x$

$-4x \quad -4x$

$x < 3$

TEST PREP

61. If $a - b > a + b$, which statement is true?

Ⓐ The value of a is positive. Ⓒ The value of a is negative.

Ⓑ The value of b is positive. Ⓓ The value of b is negative.

62. If $-a < b$, which statement is always true?

Ⓕ $a < b$ Ⓖ $a > b$ Ⓗ $a < -b$ Ⓙ $a > -b$

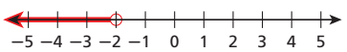
63. Which is a solution of the inequality $7(2 - x) > 4(x - 2)$?

Ⓐ -2 Ⓑ 2 Ⓒ 4 Ⓓ 7

64. Which is the graph of $-5x < -2x - 6$?

Ⓕ

Ⓗ



Ⓖ

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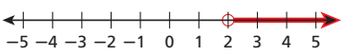


PROBLEM SOLVING

Write and solve an inequality for each situation.

1. Rosa has decided to sell pet rocks at an art fair for $5 each. She has paid $50 to rent a table at the fair and it costs her $2 to package each rock with a set of instructions. For what numbers of sales will Rosa make a profit?
   $5r > 50 + 2r$; $r > 16$
2. Jamie has a job paying $25,000 and expects to receive a $1000 raise each year. Wei has a job paying $19,000 a year and expects a $1500 raise each year. For what span of time is Jamie making more money than Wei?
   $25{,}000 + 1000y > 19{,}000 + 1500y$; $y < 12$
3. Sophia types 75 words per minute and is just starting to write a term paper. Patton already has 510 words written and types at a speed of 60 words per minute. For what numbers of minutes will Sophia have more words typed than Patton?
   $75m > 510 + 60m$; $m > 34$
4. Keith is racing his little sister Pattie and has given her a 15 foot head start. She runs 5 ft/sec and he is chasing at 8 ft/sec. For how long can Pattie stay ahead of Keith?
   $15 + 5s > 8s$; $s < 5$

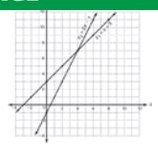
The table below shows the population of four cities in 2004 and the amount of population change from 2003. Use this table to answer questions 5–6.

City	Population (2004)	Population Change (from 2003)
Lakewood, CO	141,301	−830
Vallejo, CA	118,349	−1155
Carrollton, TX	117,823	+1170
Manchester, NH	109,310	+261

5. If the trends in this table continue, after how many years y will the population of Manchester, NH, be more than the population of Vallejo, CA? Round your answer to the nearest tenth of a year.
   A $y > 0.2$ C $y > 34.6$
   B $y > 6.4$ D $y > 78.6$
6. If the trends in this table continue, for how long x will the population of Carrollton, TX be less than the population of Lakewood, CO? Round your answer to the nearest tenth of a year.
   F $x < 11.7$ H $x < 20.1$
   G $x < 14.6$ J $x < 28.3$

CHALLENGE

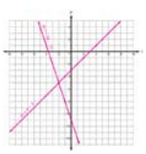
The grid at right shows the graphs of two functions, $y_1 = x + 3$ and $y_2 = 2x - 1$. These functions can be graphed by creating a table of ordered pairs for each function and then plotting the ordered pairs. The ordered pairs of each function form straight lines as shown.



1. For what value of x is $y_1 = y_2$? What happens to the graphs at this x-value?
   $x = 4$; the lines intersect
2. For what values of x is $y_1 > y_2$? What happens to the graphs for these x-values?
   $x < 4$; the line for y_1 is above (greater than) the line for y_2
3. The equation $y_1 = x + 3$ tells you that y_1 is the same as the expression $x + 3$. Likewise, $y_2 = 2x - 1$ tells you that y_2 is the same as $2x - 1$. Use this information to rewrite $y_1 > y_2$ using the variable x.
   $y_1 > y_2$
   $x + 3 > 2x - 1$
4. Solve the inequality in problem 3. How do the solutions relate to your answer for problem 2?
   $x < 4$; same solutions as problem 2
5. Explain how you could use the graph to solve this inequality: $2x - 1 \geq x + 3$.
   Possible answer: Find the x-values for which the line for y_2 is either above (greater than) or intersects (equal to) the line for y_1. The solutions are $x \geq 4$.
6. Generate ordered pairs for the functions below. For each, plot the points, connect them with straight lines, and label as in the example above.

$y_1 = x - 3$	
x	y
−4	−7
−2	−5
0	−3
2	−1
4	1

$y_2 = -3x - 11$	
x	y
−4	1
−3	−2
−2	−5
−1	−8
0	−11



Use the graph to solve $x - 3 \leq -3x - 11$.
$x \leq -2$

Teaching Tip **Geometry** For **Exercise 57**, the largest angle is already greater than either of the other two, so students only need to check if the sum of the measures of the two smaller angles (x and $x - 1$) is greater than the measure of the largest angle ($2x$). The problem could also be solved by writing and solving an equation. Set the sum of the angle measures equal to 180, solve for x, and then substitute x back into each expression. One angle is greater than 90°, so it is not acute.

TEST PREP DOCTOR In **Exercise 61,** tell students to solve for either variable. Both will result in the inequality $b < 0$.

In **Exercise 63,** students who chose **C** or **D** may not have changed the inequality sign when dividing both sides by a negative number. Students who chose **B** chose the endpoint, which is not included in the solutions.

Answers

57. No; the sum of the angle measures x and $x - 1$ would need to be greater than $2x$.
$x + (x - 1) > 2x$:
$2x - 1 > 2x$ has no solutions.

58. The steps are identical except when you multiply or divide both sides of an inequality by a negative number, when you must reverse the inequality symbol.

65. Short Response Write a real-world situation that could be modeled by the inequality $7x + 4 > 4x + 13$. Explain how the inequality relates to your situation.

Possible answer: parking lot A starts with 4 cars and 7 more cars park each hour. Parking lot B starts with 13 cars and 4 more cars park each hour. The inequality helps you find that parking lot A has more cars than parking lot B after 3 hours.

CHALLENGE AND EXTEND

Solve each inequality.

66. $2\frac{1}{2} + 2x \geq 5\frac{1}{2} + 2\frac{1}{2}x$ $x \leq -6$

67. $1.6x - 20.7 > 6.3x - (-2.2x)$ $x < -3$

68. $1.3x - 7.5x < 8.5x - 29.4$ $x > 2$

69. $-4w + \frac{-8 - 37}{9} \leq \frac{75 - 3}{9} + 3w$ $w \geq -1\frac{6}{7}$

70. Replace the square and circle with numbers so that the inequality has all real numbers as solutions. $\square - 2x < \bigcirc - 2x$ Check students' work: the number in the square should be less than the number in the circle.

71. Replace the square and circle with numbers so that the inequality has no solutions. $\square - 2x < \bigcirc - 2x$ Check students' work: the number in the square should be greater than the number in the circle.

72. Critical Thinking Explain whether there are any numbers that can replace the square and circle so that the inequality has all real numbers as solutions. $\square + 2x < \bigcirc + x$ No; solving gives $x <$ [○ – □] where [○ – □] is a real number. So no real number greater than or equal to [○ – □] is a solution.

Career Path

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Katie Flannigan
Culinary Arts program

Q: What math classes did you take in high school?
A: Algebra 1, Geometry, and Algebra 2

Q: What math classes have you taken since high school?
A: I have taken a basic accounting class and a business math class.

Q: How do you use math?
A: I use math to estimate how much food I need to buy. I also use math when adjusting recipe amounts to feed large groups of people.

Q: What are your future plans?
A: I plan to start my own catering business. The math classes I took will help me manage the financial aspects of my business.

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Journal

Given $n \geq 4n + 9$, have students explain how they decide on which side of the inequality to gather the variable terms and why. Then have students solve the inequality, explaining each step.

ALTERNATIVE ASSESSMENT

Have students write, solve, and graph an inequality that has variables on both sides and that needs to be simplified before applying inverse operations.

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2-5 Lesson Quiz

Solve each inequality and graph the solutions.

1. $t < 5t + 24$ $t > -6$

2. $5x - 9 \leq 4.1x - 81$
$x \leq -80$

3. $4b + 4(1 - b) > b - 9$
$b < 13$

4. Rick bought a photo printer and supplies for $186.90, which will allow him to print photos for $0.29 each. A photo store charges $0.55 to print each photo. How many photos must Rick print before his total cost is less than getting prints made at the photo store? more than 718 photos

Solve each inequality.

5. $2y - 2 \geq 2(y + 7)$
no solutions

6. $2(-6r - 5) < -3(4r + 2)$
all real numbers

Also available online

2-6 Algebra LAB

Truth Tables and Compound Statements

Use with Solving Compound Inequalities

A compound statement is formed by combining two or more simple statements. A compound statement is either true or false depending on whether its simple statements are true or false.

MATHEMATICAL PRACTICES **Look for and express regularity in repeated reasoning.**

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Activity 1

• Let *P* be "Cindy is at least 17 years old." • Let *Q* be "Cindy has a driver's license."

If...	then *P* is	and *Q* is	so *P* AND *Q* is
Cindy is 18 years old. Cindy has a driver's license.	True	True	True
Cindy is 17 years old. Cindy does not have a driver's license.	True	False	False
Cindy is 16 years old. Cindy has a driver's license.	False	True	False
Cindy is 15 years old. Cindy does not have a driver's license.	False	False	False

P **AND** *Q* is true when _____?_____. **both *P* AND *Q* are true**

Try This

For each pair of simple statements, tell whether *P* AND *Q* is true or false.

1. *P:* Many birds can fly; *Q:* A zebra is an animal. ***P* AND *Q* is true.**

Activity 2

• Let *P* be "Paul plays tennis." • Let *Q* be "Paul has brown eyes."

If...	then *P* is	and *Q* is	so *P* OR *Q* is
Paul plays tennis. Paul has brown eyes.	True	True	True
Paul plays tennis. Paul has green eyes.	True	False	True
Paul does not play tennis. Paul has brown eyes.	False	True	True
Paul does not play tennis. Paul has green eyes.	False	False	False

P **OR** *Q* is true when _____?_____. **either *P* is true or *Q* is true**

Try This

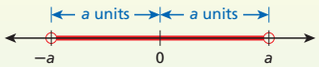
For each pair of simple statements, tell whether *P* OR *Q* is true or false.

2. *P:* The number 12 is even; *Q:* The number 12 is a composite number. ***P* OR *Q* is true.**

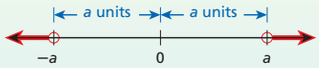
Teacher to Teacher

I usually introduce the topic of truth tables with AND or OR sentences when working with inequalities and absolute values. Students learn that "the absolute value of an expression is less than a number" is an AND statement, and "the absolute value of an expression is greater than a number" is an OR statement.

$|x| < a$ (where $a > 0$)
$x > -a$ AND $x < a$



$|x| > a$ (where $a > 0$)
$x < -a$ OR $x > a$



John Remensky
Library, PA

Algebra LAB Organizer

Use with the lesson *Solving Compound Inequalities*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Determine whether compound statements are true or false.

INTERACTIVE **Online Edition**

Countdown

Resources

LAB ***Algebra Lab Activities***
Lab Recording Sheet

Teach

Discuss

P and *Q* are variables that are replaced by phrases, rather than by numbers.

Teaching Tip **Math Background**
P AND *Q* can be written as $P \wedge Q$. *P* OR *Q* can be written as $P \vee Q$.

Close

Key Concept

An AND statement is true if *both* statements are true. An OR statement is true if *at least one* of the statements is true.

Assessment

Journal Have students explain how to determine if a compound statement with AND is true, and how to determine if a compound statement with OR is true. Explanations should include examples.

MATHEMATICAL PRACTICES

Look for and express regularity in repeated reasoning.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online
State Resources Online

2-6 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Solve compound inequalities in one variable.

Graph solution sets of compound inequalities in one variable.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each inequality.

1. $x + 3 \leq 10$ $x \leq 7$

2. $23 < -2x + 3$ $-10 > x$

Solve each inequality and graph the solutions.

3. $4x + 1 \leq 25$ $x \leq 6$

4. $0 \geq 3x + 3$ $-1 \geq x$

Also available online

Math Humor

Q: How does a math teacher get a compound fracture?

A: She breaks her hAND.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online
State Resources Online

COMMON CORE

2-6 Solving Compound Inequalities

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Objectives
Solve compound inequalities in one variable.

Graph solution sets of compound inequalities in one variable.

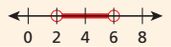
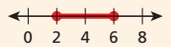
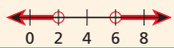
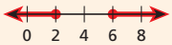
Vocabulary
compound inequality
intersection
union

Who uses this?
A lifeguard can use compound inequalities to describe the safe pH levels in a swimming pool. (See Example 1.)

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a **compound inequality**.

Know it! Note

Compound Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers greater than 2 AND less than 6	$x > 2$ AND $x < 6$ $2 < x < 6$	
All real numbers greater than or equal to 2 AND less than or equal to 6	$x \geq 2$ AND $x \leq 6$ $2 \leq x \leq 6$	
All real numbers less than 2 OR greater than 6	$x < 2$ OR $x > 6$	
All real numbers less than or equal to 2 OR greater than or equal to 6	$x \leq 2$ OR $x \geq 6$	

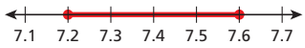
EXAMPLE 1 ***Chemistry Application***

A water analyst recommends that the pH level of swimming pool water be between 7.2 and 7.6 inclusive. Write a compound inequality to show the pH levels that are within the recommended range. Graph the solutions.

Let p be the pH level of swimming pool water.

7.2	is less than or equal to	pH level	is less than or equal to	7.6
7.2	≤	p	≤	7.6

$7.2 \leq p \leq 7.6$



Helpful Hint
The phrase "between 7.2 and 7.6 *inclusive*" means that the numbers 7.2 and 7.6 are included in the solutions. Use a solid circle for endpoints that are solutions.

(br), Sam Dudgeon/HMH; (tl), Design Pics/Carson Ganci/Getty Images

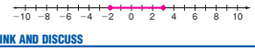
1 Introduce

EXPLORATION

Solving Compound Inequalities

Using a graphing calculator, follow the steps below to find the values of x when $2x + 1$ is between -3 and 7.

1. Press Y=. Enter **2X + 1** for **Y1**, **−3** for **Y2**, and **7** for **Y3**.
2. Press GRAPH to graph the functions.
3. Press TRACE and use the arrow keys to move the cursor. Find the x-values where the graph of $y = 2x + 1$ crosses the horizontal lines.
4. For which values of x is $2x + 1$ between -3 and 7?
5. Press 2nd TABLE to examine these functions in a table. For which values of x is $2x + 1$ between -3 and 7? Graph these values on a number line.



THINK AND DISCUSS

6. **Explain** how the solution to this problem is different from the solutions to problems you studied earlier in the chapter.
7. **Explain** how the graph of this solution looks different from the graphs you studied earlier in the chapter.

3. It crosses $y = -3$ when $x = -2$ and crosses $y = 7$ when $x = 3$.
4. for all x-values between -2 and 3
5. for all x-values between -2 and 3
6. Earlier, the solutions were the set of all numbers greater than or less than a number. In

Motivate

Read the sentences and follow-up questions below.

Max will take his brother or sister to the concert.

How many people will Max take? 1 How do you know? the word *or*

Max will take his brother and sister to the concert.

How many people will Max take? 2 How do you know? the word *and*

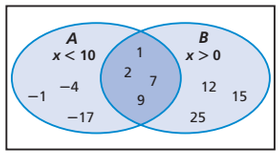
The words *or* and *and* can be used to represent the solutions of a compound inequality.

Explorations and answers are provided in your online resources.

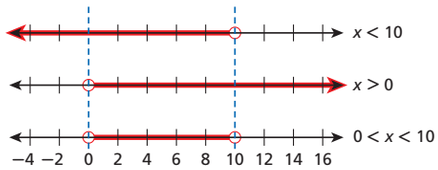
CHECK IT OUT! **1.** The free chlorine level in a pool should be between 1.0 and 3.0 parts per million inclusive. Write a compound inequality to show the levels that are within this range. Graph the solutions.

$1.0 \le c \le 3.0$

In this diagram, oval A represents some integer solutions of $x < 10$, and oval B represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of *both* $x < 10$ *and* $x > 0$.



You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the **intersection** and shows the numbers that are solutions of both inequalities.



EXAMPLE 2 Solving Compound Inequalities Involving AND

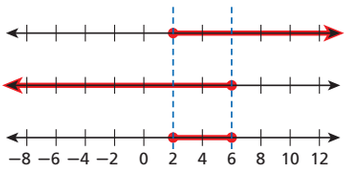
Solve each compound inequality and graph the solutions.

A $4 \le x + 2 \le 8$

$4 \le x + 2$ AND $x + 2 \le 8$ *Write the compound inequality using AND.*

$-2 \quad -2 \qquad -2 \quad -2$ *Solve each simple inequality.*

$2 \le x$ AND $x \le 6$



Graph $2 \le x$.

Graph $x \le 6$.

Graph the intersection by finding where the two graphs overlap.

B $-5 \le 2x + 3 < 9$

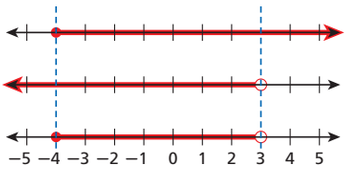
$-5 \le 2x + 3 < 9$ *Since 3 is added to 2x, subtract 3 from each part of the inequality.*

$-3 \quad -3 \quad -3$

$-8 \le 2x < 6$

$\frac{-8}{2} \le \frac{2x}{2} < \frac{6}{2}$ *Since x is multiplied by 2, divide each part of the inequality by 2.*

$-4 \le x < 3$



Graph $-4 \le x$.

Graph $x < 3$.

Graph the intersection by finding where the two graphs overlap.

Remember!

The statement $-5 \le 2x + 3 \le 9$ consists of two inequalities connected by AND. Example 2B shows a "shorthand" method for solving this type of inequality.

2a. $1 < x < 5$

2b. $-3 \le n < 2$

CHECK IT OUT! **Solve each compound inequality and graph the solutions.**

2a. $-9 < x - 10 < -5$ **2b.** $-4 \le 3n + 5 < 11$

COMMON ERROR ALERT

Students may be confused when using the shorthand method for AND in **Example 2B.** When reading the expression $a < x < b$, students can say, "x is greater than a AND x is less than b." Students can also say, "a is less than x AND x is less than b." This shorthand method for writing a compound inequality exists only for the AND statement.

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Additional Examples

Example 1

The pH level of a popular shampoo is between 6.0 and 6.5 inclusive. Write a compound inequality to show the pH levels of this shampoo. Graph the solutions. $6.0 \le p \le 6.5$

Example 2

Solve each compound inequality and graph the solutions.

A. $-5 < x + 1 < 2$ $-6 < x < 1$

B. $8 < 3x - 1 \le 11$ $3 < x \le 4$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How would the solution change if the range of given values were not inclusive?

EXAMPLE 2

- How do you write a compound inequality as two simple inequalities?

2 Teach

Guided Instruction

Review solving multi-step inequalities. Solve compound inequalities with AND and then OR, and then discuss the differences between the two. Help students make the connection between AND and intersection and between OR and union.

Reaching All Learners

Through Kinesthetic Experience

Assign each student a number. (Assign 1–20 to twenty students.) Then have students stand when you read statements that describe their number. For instance:

- greater than 15 OR less than 2;
- less than 12 AND greater than or equal to 8;
- greater than or equal to 3 AND less than or equal to 3;
- greater than 2 OR less than 3

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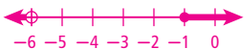
Additional Examples

Example 3

Solve each compound inequality and graph the solutions.

A. $8 + t \geq 7$ OR $8 + t < 2$

$t \geq -1$ OR $t < -6$



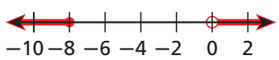
B. $4x \leq 20$ OR $3x > 21$

$x \leq 5$ OR $x > 7$

Example 4

Write the compound inequality shown by each graph.

A.



$x \leq -8$ OR $x > 0$

B.

$m > -2$ AND $m < 5$

Also available online

Questioning Strategies

EXAMPLE 3

- Describe a characteristic of the graph of the solutions of a compound inequality involving OR.

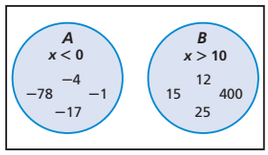
EXAMPLE 4

- How do you know whether a graph represents a compound inequality that involves AND or OR?

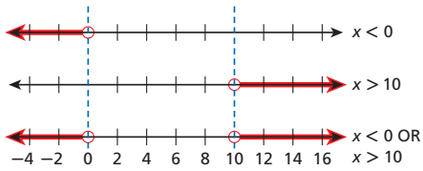
Teaching Tip **Reading Math** Discuss the everyday meanings of *intersection* and *union*. The intersection of two streets is where they cross each other. A labor union is an organization of workers who join together.

ENGLISH LANGUAGE LEARNERS

In this diagram, circle A represents some integer solutions of $x < 0$, and circle B represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of *either* $x < 0$ *or* $x > 10$.



You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combined regions are called the **union** and show the numbers that are solutions of either inequality.



EXAMPLE 3 Solving Compound Inequalities Involving OR

Solve each compound inequality and graph the solutions.

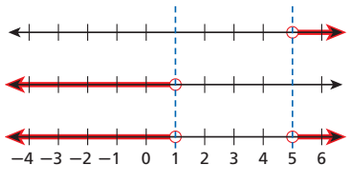
A $-4 + a > 1$ OR $-4 + a < -3$

$-4 + a > 1$ OR $-4 + a < -3$

$+4 \quad +4 \quad +4 \quad +4$

$a > 5$ OR $a < 1$

Solve each simple inequality.



Graph $a > 5$.

Graph $a < 1$.

Graph the union by combining the regions.

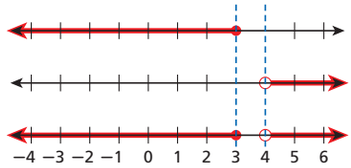
B $2x \leq 6$ OR $3x > 12$

$2x \leq 6$ OR $3x > 12$

$\frac{2x}{2} \leq \frac{6}{2} \quad \frac{3x}{3} > \frac{12}{3}$

$x \leq 3$ OR $x > 4$

Solve each simple inequality.



Graph $x \leq 3$.

Graph $x > 4$.

Graph the union by combining the regions.

3a.

3b.

CHECK IT OUT! **Solve each compound inequality and graph the solutions.**

3a. $2 + r < 12$ OR $r + 5 > 19$ $r < 10$ OR $r > 14$

3b. $7x \geq 21$ OR $2x < -2$ $x \geq 3$ OR $x < -1$

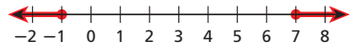
Every solution of a compound inequality involving AND must be a solution of both parts of the compound inequality. If no numbers are solutions of *both* simple inequalities, then the compound inequality has no solutions.

The solutions of a compound inequality involving OR are not always two separate sets of numbers. There may be numbers that are solutions of both parts of the compound inequality.

EXAMPLE 4 Writing a Compound Inequality from a Graph

Write the compound inequality shown by each graph.

A



The shaded portion of the graph is not between two values, so the compound inequality involves OR.

On the left, the graph shows an arrow pointing left, so use either < or ≤.
The solid circle at −1 means −1 is a solution, so use ≤.

$x \leq -1$

On the right, the graph shows an arrow pointing right, so use either > or ≥.
The solid circle at 7 means 7 is a solution, so use ≥.

$x \geq 7$

The compound inequality is $x \leq -1$ OR $x \geq 7$.

B

The shaded portion of the graph is between the values 0 and 6, so the compound inequality involves AND.

The shaded values are to the right of 0, so use > or ≥.
The solid circle at 0 means 0 is a solution, so use ≥.

$x \geq 0$

The shaded values are to the left of 6, so use < or ≤.
The empty circle at 6 means 6 is not a solution, so use <.

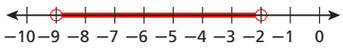
$x < 6$

The compound inequality is $x \geq 0$ AND $x < 6$.

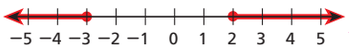
Writing Math

The compound inequality in Example 4B can also be written with the variable between the two endpoints.
$0 \leq x < 6$

CHECK IT OUT! Write the compound inequality shown by the graph.

4a. 

$-9 < y < -2$

4b. 

$x \leq -3$ OR $x \geq 2$

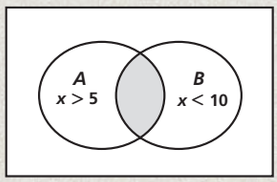
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe how to write the compound inequality $y > 4$ AND $y \leq 12$ without using the joining word AND.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizers. Write three solutions in each of the three sections of the diagram. Then write each of your nine solutions in the appropriate column or columns of the table.



$x > 5$ AND $x < 10$	$x > 5$ OR $x < 10$

Teaching Tip

Visual Use color to help students compare AND and OR statements. On a transparency, graph the solutions of $x < 3$ with a yellow marker and the solutions of $x < 7$ with a blue marker. Show that the solution set for $x < 3$ AND $x < 7$ is the green area described by $x < 3$. Then discuss the solution set for $x < 3$ OR $x < 7$. The solution is anywhere the graph is shaded, as described by $x < 7$.

Teaching Tip

Multiple Representations Sometimes a parenthesis is used for an empty circle and a square bracket is used for a solid circle. The values described by $a < x \leq b$ can also be written as $(a, b]$, a notation used more often in higher-level math. The symbols may also be used on a graph.

3 Close

Summarize

Remind students that solving compound inequalities requires solving two separate inequalities.

Emphasize that an AND statement represents an intersection, and the solutions will be those values that make both inequalities true.

An OR statement represents a union, and the solutions combine two sets of numbers, which may or may not overlap. The solutions make either inequality true.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. $y > 4$ can be written as $4 < y$. Combine the inequalities $4 < y$ and $y \leq 12$ to write $4 < y \leq 12$.
2. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 15–19, 30–35, 42
- **Average** 15–19, 30–35, 45
- **Advanced** 15–19, 30–35, 42, 43, 45, 51

If you finished Examples **1–4**
- **Basic** 15–40, 42, 45, 47–50
- **Average** 15–29, 30–40 even, 42–54
- **Advanced** 15–29, 34, 36–40 even, 42–56

Homework Quick Check
Quickly check key concepts.
Exercises: 15, 18, 20, 26, 34, 42

Answers

2.

3.

4.

5.

6.

7–10, 15–23. For graphs, see Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 2, 15, 28–29, 34–35, 42–43

Reason abstractly and quantitatively. Exercises 30–33, 55–56

Construct viable arguments and critique the reasoning of others. Exercises 44–46

Learn It Online
State Resources Online

2-6 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** The graph of a(n) ___?___ shows all values that are solutions to both simple inequalities that make a compound inequality. (*union* or *intersection*) intersection

SEE EXAMPLE 1

2. **Biology** An iguana needs to live in a warm environment. The temperature in a pet iguana's cage should be between 70 °F and 95 °F inclusive. Write a compound inequality to show the temperatures that are within the recommended range. Graph the solutions. $70 \le t \le 95$

Solve each compound inequality and graph the solutions.

SEE EXAMPLE 2

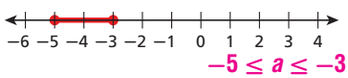
3. $-3 < x + 2 < 7$ $-5 < x < 5$
4. $5 \le 4x + 1 \le 13$ $1 \le x \le 3$
5. $2 < x + 2 < 5$ $0 < x < 3$
6. $11 < 2x + 3 < 21$ $4 < x < 9$

SEE EXAMPLE 3

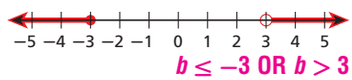
7. $x + 2 < -6$ OR $x + 2 > 6$ $x < -8$ OR $x > 4$
8. $r - 1 < 0$ OR $r - 1 > 4$ $r < 1$ OR $r > 5$
9. $n + 2 < 3$ OR $n + 3 > 7$ $n < 1$ OR $n > 4$
10. $x - 1 < -1$ OR $x - 5 > -1$ $x < 0$ OR $x > 4$

SEE EXAMPLE 4

Write the compound inequality shown by each graph.

11. 

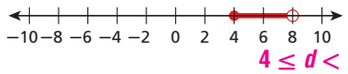
$-5 \le a \le -3$

12. 

$b \le -3$ OR $b > 3$

13.

$c < 1$ OR $c \ge 9$

14. 

$4 \le d < 8$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15	1
16–19	2
20–23	3
24–27	4

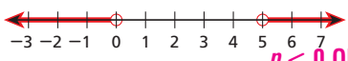
Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

15. **Meteorology** One layer of Earth's atmosphere is called the stratosphere. At one point above Earth's surface the stratosphere extends from an altitude of 16 km to an altitude of 50 km. Write a compound inequality to show the altitudes that are within the range of the stratosphere. Graph the solutions. $16 \le k \le 50$

Solve each compound inequality and graph the solutions.

16. $-1 < x + 1 < 1$ $-2 < x < 0$
17. $1 \le 2n - 5 \le 7$ $3 \le n \le 6$
18. $-2 < x - 2 < 2$ $0 < x < 4$
19. $5 < 3x - 1 < 17$ $2 < x < 6$
20. $x - 4 < -7$ OR $x + 3 > 4$ $x < -3$ OR $x > 1$
21. $2x + 1 < 1$ OR $x + 5 > 8$ $x < 0$ OR $x > 3$
22. $x + 1 < 2$ OR $x + 5 > 8$ $x < 1$ OR $x > 3$
23. $x + 3 < 0$ OR $x - 2 > 0$ $x < -3$ OR $x > 2$

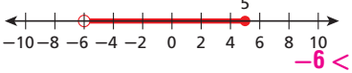
Write the compound inequality shown by each graph.

24. 

$p < 0$ OR $p > 5$

25.

$q < 0$ OR $q \ge 2$

26. 

$-6 < r \le 5$

27.

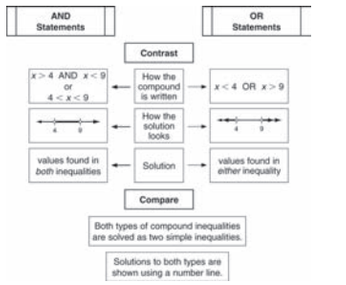
$-2 < s < 1$

28. **Music** A typical acoustic guitar has a range of three octaves. When the guitar is tuned to "concert pitch," the range of frequencies for those three octaves is between 82.4 Hz and 659.2 Hz inclusive. Write a compound inequality to show the frequencies that are within the range of a typical acoustic guitar. Graph the solutions.

28. $82.4 \le f \le 659.2$

READING STRATEGIES

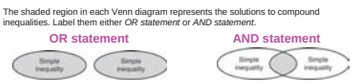
There are two types of compound inequalities: AND statements and OR statements. The chart below shows their similarities and differences.



Answer each question using the examples given above.

1. Which has more *whole number* solutions: the AND or the OR statement? OR
2. List 3 *whole number* solutions to the AND statement. Possible answer: 5, 6, 7
3. List 3 *whole number* solutions to the OR statement. Possible answer: 3, 10, 11
4. Which is best described as having solutions *between* two values: the AND statement or the OR statement? AND
5. The shaded region in each Venn diagram represents the solutions to compound inequalities. Label them either *OR statement* or *AND statement*.

OR statement AND statement



RETEACH

Compound inequalities using AND require you to find solutions so that two inequalities will be satisfied at the same time.

Solve $2 < x + 3 \le 5$ and graph the solutions.

The two inequalities are: $2 < x + 3$ AND $x + 3 \le 5$.

Solve $2 < x + 3$.

$2 < x + 3$

$-3 \quad -3$ Add -3 to both sides.

$-1 < x$

Solve $x + 3 \le 5$.

$x + 3 \le 5$

$-3 \quad -3$ Add -3 to both sides.

$x \le 2$

Graph $x > -1$.

Graph $x \le 2$.

Use **overlapping** regions for compound inequalities with **AND**.

Graph $-1 < x \le 2$.

Write the two inequalities that must be solved in order to solve each compound inequality.

1. $-3 < x - 4 \le 10$ $3 < x - 4$ AND $x - 4 \le 10$
2. $8 \le m + 4 \le 15$ $8 \le m + 4$ AND $m + 4 \le 15$
3. Graph $-2 \le w < 6$ by graphing each inequality separately. Then graph the compound inequality.

Solve each compound inequality and graph the solutions.

4. $-5 < k - 1 < 0$ $-4 < k < 1$
5. $-4 < 2x - 8 \le 6$ $2 < x \le 7$

MULTI-STEP TEST PREP

29. Jenna's band is going to record a CD at a recording studio. They will pay $225 to use the studio for one day and $80 per hour for sound technicians. Jenna has $200 and can reasonably expect to raise up to an additional $350 by taking pre-orders for the CDs.

a. Explain how the inequality $200 \le 225 + 80n \le 550$ can be used to find the number of hours Jenna and her band can afford to use the studio and sound technicians.

b. Solve the inequality. Are there any numbers in the solution set that are not reasonable in this situation? **$-0.3125 \le n \le 4.0625$; n cannot be negative**

c. Suppose Jenna raises $350 in pre-orders. How much more money would she need to raise if she wanted to use the studio and sound technicians for 6 hours? **$155**

Write and graph a compound inequality for the numbers described.

30. all real numbers between -6 and 6 **$-6 < x < 6$**

31. all real numbers less than or equal to 2 and greater than or equal to 1 **$1 \le x \le 2$**

32. all real numbers greater than 0 and less than 15 **$0 < x < 15$**

33. all real numbers between -10 and 10 inclusive **$-10 \le x \le 10$**

LINK

Chemistry

The element gallium is in a solid state at room temperature but becomes a liquid at about 30 °C. Gallium stays in a liquid state until it reaches a temperature of about 2204 °C.

34. **Transportation** The cruise-control function on Georgina's car should keep the speed of the car within 3 mi/h of the set speed. Write a compound inequality to show the acceptable speeds s if the set speed is 55 mi/h. Graph the solutions. **$52 \le x \le 58$**

35. **Chemistry** Water is not a liquid if its temperature is above 100 °C or below 0 °C. Write a compound inequality for the temperatures t when water is not a liquid. **$t < 0$ OR $t > 100$**

Solve each compound inequality and graph the solutions.

36. $5 \le 4b - 3 \le 9$ **$2 \le b \le 3$**

37. $-3 < x - 1 < 4$ **$-2 < x < 5$**

38. $r + 2 < -2$ OR $r - 2 > 2$ **$r < -4$ OR $r > 4$**

39. $2a - 5 < -5$ OR $3a - 2 > 1$ **$a < 0$ OR $a > 1$**

40. $x - 4 \ge 5$ AND $x - 4 \le 5$ **$x \ge 9$ AND $x \le 9$**

41. $n - 4 < -2$ OR $n + 1 > 6$ **$n < 2$ OR $n > 5$**

42. **Sports** The ball used in a soccer game may not weigh more than 16 ounces or less than 14 ounces at the start of the match. After $1\frac{1}{2}$ ounces of air was added to a ball, the ball was approved for use in a game. Write and solve a compound inequality to show how much the ball might have weighed before the air was added. **$12.5 \le w \le 14.5$**

43. **Meteorology** Tornado damage is rated using the Fujita scale shown in the table. A tornado has a wind speed of 200 miles per hour. Write and solve a compound inequality to show how many miles per hour the wind speed would need to increase for the tornado to be rated "devastating" but not "incredible." **$7 \le m \le 60$**

Fujita Tornado Scale

Category	Type	Wind Speed (mi/h)
F0	Weak	40 to 72
F1	Moderate	73 to 112
F2	Significant	113 to 157
F3	Severe	158 to 206
F4	Devastating	207 to 260
F5	Incredible	261 to 318

44. Give a real-world situation that can be described by a compound inequality. Write the inequality that describes your situation.

44. Possible answer: Margaret is expecting between 25 and 35 guests; $25 \le g \le 35$ where g is a natural number

45. **Write About It** How are the graphs of the compound inequality $x < 3$ AND $x < 7$ and the compound inequality $x < 3$ OR $x < 7$ different? How are the graphs alike? Explain.

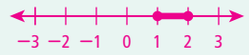
(t)b, © Brand X Pictures; (c)Kenneth Eward/Fundamental Photographs/Photo Researchers

MULTI-STEP TEST PREP **Exercise 29** involves using inequalities to find the number of hours a band can afford to rent a recording studio. This exercise prepares students for the Multi-Step Test Prep.

Answers

29a. $225 + 80n$ gives the cost of the studio and technicians. The band will spend between $200 and $550.

30.

31.


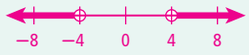
32.

33.

34.

36.

37.

38.


39.

40.

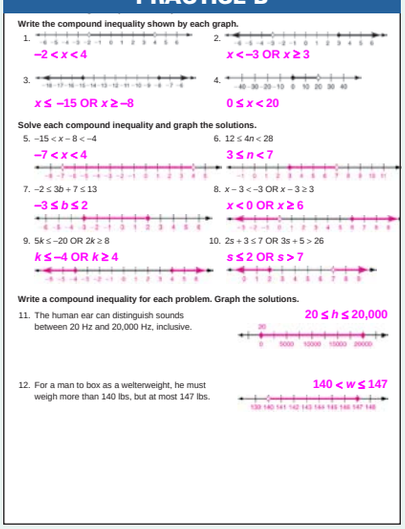
41.

45. Both graphs include numbers less than 3. The graph of $x < 3$ AND $x < 7$ does not include numbers greater than or equal to 3. The graph of $x < 3$ OR $x < 7$ also includes all numbers less than 7.

PRACTICE A

PRACTICE C

PRACTICE B


Write the compound inequality shown by each graph.

1. $-2 < x < 4$
2. $x < -3$ OR $x \ge 3$
3. $x \le -15$ OR $x \ge -8$
4. $0 \le x < 20$

Solve each compound inequality and graph the solutions.

5. $-15 < x - 8 < -4$ → $-7 < x < 4$
6. $12 \le 4n < 28$ → $3 \le n < 7$
7. $-2 \le 3b + 7 \le 13$ → $-3 \le b \le 2$
8. $x - 3 < -3$ OR $x - 3 \ge 3$ → $x < 0$ OR $x \ge 6$
9. $5k \le -20$ OR $2k \ge 8$ → $k \le -4$ OR $k \ge 4$
10. $2s + 3 \le 7$ OR $3s + 5 > 26$ → $s \le 2$ OR $s > 7$

Write a compound inequality for each problem. Graph the solutions.

11. The human ear can distinguish sounds between 20 Hz and 20,000 Hz, inclusive. **$20 \le h \le 20{,}000$**
12. For a man to box as a welterweight, he must weigh more than 140 lbs, but at most 147 lbs. **$140 < w \le 147$**

PROBLEM SOLVING

Write and solve an inequality for each situation.

1. The Mexican Tetra is a tropical fish that requires a water temperature between 68 and 77 degrees Fahrenheit, inclusive. An aquarium is heated 8 degrees so that a Tetra can live in it. What temperatures could the water have been before the heating? **$68 \le t + 8 \le 77$; $60 \le t \le 69$**
2. Nerissa's car can travel between 380 and 410 miles on a full tank of gas. She filled her gas tank and drove 45 miles. How many more miles can she drive without running out of gas? **$380 \le m + 45 \le 410$; $335 \le m \le 365$**
3. A local company is hiring trainees with less than 1 year of experience and managers with 5 or more years of experience. Graph the solutions. **$y < 1$ OR $y \ge 5$**
4. Marty's allowance is doubled and is now between $10 and $15, inclusive. What amounts could his allowance have been before the increase? Graph the solutions. **$10 \le 2a \le 15$; $5 \le a \le 7.5$**

The elliptical orbits of planets bring them closer to and farther from the Sun at different times. The closest (perihelion) and furthest (aphelion) points are given for three planets below. Use this data to answer questions 5–7.

Planet	Perihelion (in 10^6 km)	Aphelion (in 10^6 km)
Uranus	2741.3	3003.6
Neptune	4444.5	4545.7
Pluto	4435.0	7304.3

5. Which inequality represents the distances d from the sun to Neptune?
   A $d \le 4444.5$
   B $d \le 4545.7$
   C $4444.5 \le d \le 4545.7$
   D $d = 4444.5$ OR $d \ge 4545.7$
6. A NASA probe is traveling between Uranus and Neptune. It is currently between their orbits. Which inequality shows the possible distance p from the probe to the Sun?
   F $1542.1 < p < 1703.2$
   G $2741.3 < p < 4545.7$
   H $3003.6 < p < 4444.5$
   J $7185.8 < p < 7549.3$
7. At what distances o do the orbits of Neptune and Pluto overlap?
   A $4435.0 \le o \le 4444.5$
   B $4435.0 \le o \le 4545.7$
   C $4444.5 \le o \le 7304.3$
   D $4545.7 \le o \le 7304.3$

CHALLENGE

The inequality $a \le x \le b$, where $a < b$, is the set of all real numbers between a and b, including a and b. The corresponding set of points on the number line is called a *closed interval*.

In Exercises 1–7, n is an integer and $n - \frac{1}{4} \le x \le n + \frac{1}{4}$. Graph the solutions for the given values of n on the number line below.

1. $n = 0$
2. $n = 0$ or $n = 1$
3. $n = 0$ or $n = -1$
4. $n = -2, -1, 0, 1$, or 2
5. $n = -3, -2, -1, 0, 1, 2$, or 3
6. $n = -4, -3, -2, -1, 0, 1, 2, 3$, or 4

7. Suppose that the graphs for all values of n are shown on the number line. Describe the graph.
   Answers may vary. Sample answer: closed intervals centered at each integer with each interval being $\frac{1}{2}$ unit long

Let n be an integer. For each set of intervals on the given number line, write a compound inequality involving n, x, and a conjunction to describe the entire graph.

8. **$n \le x \le n + \frac{1}{2}$**
9. **$2n \le x \le 2n + 1$**
10. **$4n - 1 \le x \le 4n + 1$**
11. a. Let n be a nonzero integer. Describe the interval $1 - \frac{1}{n} \le x \le 1 + \frac{1}{n}$.
    Answers may vary. Sample answers: The center of the interval is 1, and its length is $\frac{2}{n}$.
    b. What can you say about the interval as n gets larger?
    As n gets larger, the center remains at 1, but the length of the interval gets smaller.

TEST PREP DOCTOR In **Exercise 48,** students who chose **F** solved only the second inequality. Students who chose **J** graphed the solutions as an OR statement.

In **Exercise 49,** students can start by looking at the open circle at 2 and determining that the correct symbol is <. This immediately eliminates all choices but **B.**

Journal

Explain the difference between graphing an inequality involving AND and graphing an inequality involving OR.

ALTERNATIVE ASSESSMENT

Have students write and graph a compound inequality with AND and one with OR, such that solving each simple inequality involves two steps.

Power Presentations with PowerPoint®

2-6 Lesson Quiz

1. The target heart rate during exercise for a 15-year-old is between 154 and 174 beats per minute inclusive. Write a compound inequality to show the heart rates that are within the target range. Graph the solutions. $154 \le h \le 174$

Solve each compound inequality and graph the solutions.

2. $2 \le 2w + 4 \le 12$

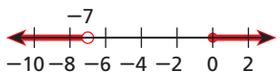
$-1 \le w \le 4$

3. $3 + r > -2$ OR $3 + r < -7$

$r > -5$ OR $r < -10$

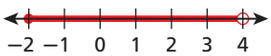
Write the compound inequality shown by each graph.

4.



$x < -7$ OR $x \ge 0$

5.



$-2 \le a < 4$

Also available online

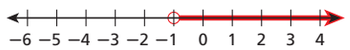
46. Critical Thinking If there is no solution to a compound inequality, does the compound inequality involve OR or AND? Explain. **An inequality with OR always has solutions because the two simple inequalities do not have to be true at the same time. A compound inequality with no solutions must involve AND.**

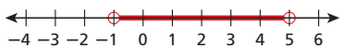
TEST PREP

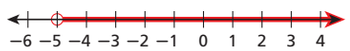
47. Which of the following describes the solutions of $-x + 1 > 2$ OR $x - 1 > 2$?

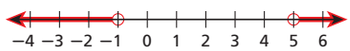
Ⓐ all real numbers greater than 1 or less than 3
Ⓑ all real numbers greater than 3 or less than 1
Ⓒ all real numbers greater than −1 or less than 3
Ⓓ all real numbers greater than 3 or less than −1

48. Which of the following is a graph of the solutions of $x - 3 < 2$ AND $x + 3 > 2$?

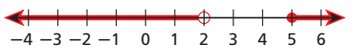
Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

49. Which compound inequality is shown by the graph?



Ⓐ $x \le 2$ OR $x > 5$
Ⓑ $x < 2$ OR $x \ge 5$
Ⓒ $x \le 2$ OR $x \ge 5$
Ⓓ $x \ge 2$ OR $x > 5$

50. Which of the following is a solution of $x + 1 \ge 3$ AND $x + 1 \le 3$?

Ⓕ 0 Ⓖ 1 Ⓗ 2 Ⓙ 3

CHALLENGE AND EXTEND

Solve and graph each compound inequality.

51. $2c - 10 < 5 - 3c < 7c$ **$0.5 < c < 3$**

52. $5p - 10 < p + 6 < 3p$ **$3 < p < 4$**

53. $2s \le 18 - s$ OR $5s \ge s + 36$ **$s \le 6$ OR $s \ge 9$**

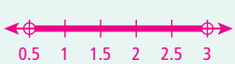
54. $9 - x \ge 5x$ OR $20 - 3x \le 17$ **$x \le 1\frac{1}{2}$ OR $x \ge 1$**

55. Write a compound inequality that represents all values of x that are NOT solutions to $x < -1$ OR $x > 3$. **$-1 \le x \le 3$**

56. For the compound inequality $x + 2 \ge a$ AND $x - 7 \le b$, find values of a and b for which the only solution is $x = 1$. **$a = 3$; $b = -6$**

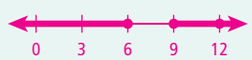
Answers

51.



52.

53.



54.

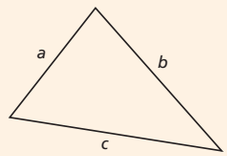
Triangle Inequality

For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

The sides of this triangle are labeled *a*, *b*, and *c*. You can use the Triangle Inequality to write three statements about the triangle.

$a + b > c$ $\quad a + c > b$ $\quad b + c > a$

Unless all three of the inequalities are true, the lengths *a*, *b*, and *c* cannot form a triangle.



Example 1

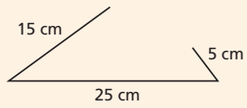
Can three side lengths of 25 cm, 15 cm, and 5 cm form a triangle?

a. $25 + 15 > 5$
$40 > 5$ *True*

b. $25 + 5 > 15$
$30 > 15$ *True*

c. $15 + 5 > 25$
$20 > 25$ *False*

One of the inequalities is false, so the three lengths will not make a triangle. The situation is shown in the figure to the right.



Example 2

Two sides of a triangle measure 8 ft and 10 ft. What is the range of lengths of the third side?

Start by writing three statements about the triangle. Use *x* for the unknown side length.

a. $8 + 10 > x$
$18 > x$
The third side must be shorter than 18 ft.

b. $8 + x > 10$
$8 + x - 8 > 10 - 8$
$x > 2$
The third side must be longer than 2 ft.

c. $x + 10 > 8$
$x + 10 - 10 > 8 - 10$
$x > -2$
This provides no new useful information.

From part **a,** the third side must be shorter than 18 ft. And from part **b,** it must be longer than 2 ft. An inequality showing this is $2 < x < 18$.

Try This

Decide whether the three lengths given can form a triangle. If not, explain.

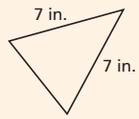
1. 14 ft, 30 ft, 10 ft
no; 14 ft + 10 ft < 30 ft

2. 11 cm, 8 cm, 17 cm
yes

3. $6\frac{1}{2}$ yd, 3 yd, $2\frac{3}{4}$ yd
no; 3 yd + $2\frac{3}{4}$ yd < $6\frac{1}{2}$ yd

Write a compound inequality for the range of lengths of the third side of each triangle.

4.



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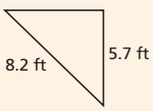

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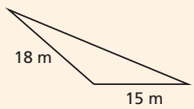


Organizer

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Apply the algebra skills of solving inequalities to finding possible side lengths of triangles.

Online Edition

Countdown

Teach

Remember

Students review and apply the geometric properties of triangles.

Kinesthetic Students may better understand the Triangle Inequality if they attempt to make triangles from various lengths of straw or spaghetti.

Close

Assess

Have students give three possible side lengths for the third side of the triangles in **Problems 4–6.**

Answers

4. 0 in. < *x* < 14 in.
5. 2.5 ft < *x* < 13.9 ft
6. 3 m < *x* < 33 m

Learn It Online
State Resources Online

2-7 Organizer

Pacing: Traditional 2 days
Block 1 day

Objective: Solve inequalities in one variable involving absolute-value expressions.

Online Edition
Tutorial Videos

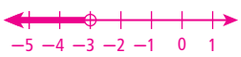
Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each inequality and graph the solutions.

1. $x + 7 < 4$ $x < -3$



2. $14x \geq 28$ $x \geq 2$

3. $5 + 2x > 1$ $x > -2$

Also available online

Math Humor

Q: What does an absolute-value expression work on when it goes to the gym?

A: Its "abs"!

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

Learn It Online
State Resources Online

COMMON CORE

2-7 Solving Absolute-Value Inequalities

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Also* **CC.9-12.A.CED.1***

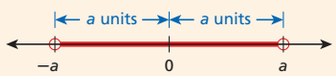
Objective
Solve inequalities in one variable involving absolute-value expressions.

Why learn this?
You can solve an absolute-value inequality to determine the safe range for the pressure of a fire extinguisher. (See Example 3.)

When an inequality contains an absolute-value expression, it can be rewritten as a compound inequality. The inequality $|x| < 5$ describes all real numbers whose distance from 0 is less than 5 units. The solutions are all numbers between -5 and 5, so $|x| < 5$ can be rewritten as $-5 < x < 5$ or as $x > -5$ AND $x < 5$.

Know it! Note

Absolute-Value Inequalities Involving <

WORDS	NUMBERS
The inequality $\|x\| < a$ (when $a > 0$) asks, "What values of x have an absolute value less than a?" The solutions are numbers between $-a$ and a.	$\|x\| < 5$ $-5 < x < 5$ $x > -5$ AND $x < 5$
GRAPH	**ALGEBRA**
	$\|x\| < a$ (when $a > 0$) $-a < x < a$ $x > -a$ AND $x < a$

The same properties are true for inequalities that use the symbol ≤.

Helpful Hint

Just as you do when solving absolute-value equations, you first isolate the absolute-value expression when solving absolute-value inequalities.

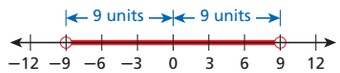
EXAMPLE 1 Solving Absolute-Value Inequalities Involving <

Solve each inequality and graph the solutions.

A $|x| + 3 < 12$

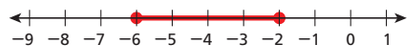
$|x| + 3 < 12$
$-3 \quad -3$
$|x| < 9$ — *Since 3 is added to $|x|$, subtract 3 from both sides to undo the addition.*

$x > -9$ AND $x < 9$ — *Write as a compound inequality.*

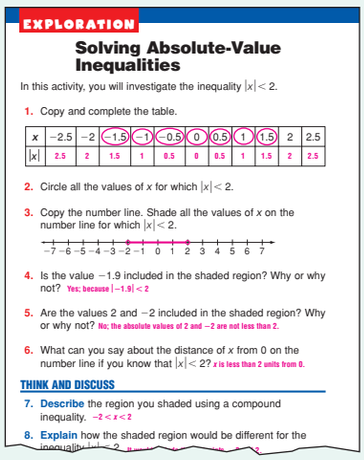


B $|x + 4| \leq 2$

$x + 4 \geq -2$ AND $x + 4 \leq 2$ — *Write as a compound inequality.*
$-4 \quad -4 \qquad -4 \quad -4$ — *Solve each inequality.*
$x \geq -6$ AND $x \leq -2$ — *Write as a compound inequality.*



1 Introduce



EXPLORATION

Solving Absolute-Value Inequalities

In this activity, you will investigate the inequality $|x| < 2$.

1. Copy and complete the table.

x	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5
$\|x\|$	2.5	2	1.5	1	0.5	0	0.5	1	1.5	2	2.5

2. Circle all the values of x for which $|x| < 2$.

3. Copy the number line. Shade all the values of x on the number line for which $|x| < 2$.

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

4. Is the value −1.9 included in the shaded region? Why or why not? Yes; because $|-1.9| < 2$

5. Are the values 2 and −2 included in the shaded region? Why or why not? No; the absolute values of 2 and −2 are not less than 2.

6. What can you say about the distance of x from 0 on the number line if you know that $|x| < 2$? x is less than 2 units from 0.

THINK AND DISCUSS

7. **Describe** the region you shaded using a compound inequality. $-2 < x < 2$

8. **Explain** how the shaded region would be different for the inequality $|x| < 2$

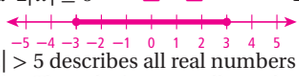
Motivate

Present the following situation to the class: To get a B on an exam, a student's score must be no more than 5 points away from 85. Ask students what scores result in a B. Plot these values on a number line. Then explain that an absolute-value inequality can be used to describe the set of scores.

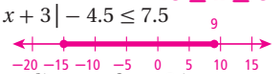
Explorations and answers are provided in your online resources.

CHECK IT OUT! Solve each inequality and graph the solutions.

1a. $2|x| \le 6$ $-3 \le x \le 3$



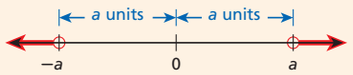
1b. $|x + 3| - 4.5 \le 7.5$ $-15 \le x \le 9$



The inequality $|x| > 5$ describes all real numbers whose distance from 0 is greater than 5 units. The solutions are all numbers less than -5 or greater than 5. The inequality $|x| > 5$ can be rewritten as the compound inequality $x < -5$ OR $x > 5$.

Know it! Note

Absolute-Value Inequalities Involving >

WORDS	NUMBERS
The inequality $\|x\| > a$ (when $a > 0$) asks, "What values of x have an absolute value greater than a?" The solutions are numbers less than $-a$ or greater than a.	$\|x\| > 5$ $x < -5$ OR $x > 5$
GRAPH	**ALGEBRA**
	$\|x\| > a$ (when $a > 0$) $x < -a$ OR $x > a$

The same properties are true for inequalities that use the symbol ≥.

EXAMPLE 2 Solving Absolute-Value Inequalities Involving >

Solve each inequality and graph the solutions.

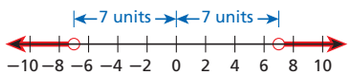
A $|x| - 20 > -13$

$|x| - 20 > -13$

$+20 \quad +20$

$|x| > 7$ *Since 20 is subtracted from $|x|$, add 20 to both sides to undo the subtraction.*

$x < -7$ OR $x > 7$ *Write as a compound inequality.*



B $|x - 8| + 5 \ge 11$

$|x - 8| + 5 \ge 11$

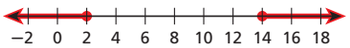
$-5 \quad -5$

$|x - 8| \ge 6$ *Since 5 is added to $|x - 8|$, subtract 5 from both sides to undo the addition.*

$x - 8 \le -6$ OR $x - 8 \ge 6$ *Write as a compound inequality.*

$+8 \quad +8 \qquad +8 \quad +8$

$x \le 2$ OR $x \ge 14$ *Solve each inequality. Write as a compound inequality.*



CHECK IT OUT! Solve each inequality and graph the solutions.

2a. $|x| + 10 \ge 12$

2b. $\left|x + 2\frac{1}{2}\right| + \frac{1}{2} \ge 4$

2a. $x \le -2$ OR $x \ge 2$

2b. $x \le -6$ OR $x \ge 1$

2 Teach

Guided Instruction

Remind students that the absolute value of a number is its distance from 0 on a number line. Then briefly review how to solve compound inequalities.

Use number lines and simple absolute-value inequalities like $|x| < 5$ and $|x| > 5$ to show why less-than symbols indicate an AND statement and greater-than symbols indicate an OR statement.

Reaching All Learners

Through Auditory Cues

Stress the word *than* when saying "less than" and stress the suffix *-er* when saying "greater." Then the appropriate compound word almost rhymes with the inequality in the following phrase: "Less than is AND, greater is OR."

COMMON ERROR ALERT

In **Example 1B,** students may try to subtract 4 from both sides of the inequality as a first step. Remind students that they must first remove the absolute-value symbol by writing a compound inequality.

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Additional Examples

Example 1

Solve each inequality and graph the solutions.

A. $|x| - 3 < -1$ $-2 < x < 2$

B. $|x - 1| \le 2$ $-1 \le x \le 3$

Example 2

Solve each inequality and graph the solutions.

A. $|x| + 14 \ge 19$ $x \le -5$ OR $x \ge 5$

B. $3 + |x + 2| > 5$ $x < -4$ OR $x > 0$

Also available online

Questioning Strategies

EXAMPLE 1

- Why does an absolute-value inequality with a less-than symbol indicate an AND statement?

EXAMPLE 2

- Why does an absolute-value inequality with a greater-than symbol indicate an OR statement?

Teaching Tip **Reading Math** Have students read $|x|$ as "the distance from zero" to help them visualize what the graph of an absolute-value inequality will look like. For example, $|x| < 3$ can be read as "the distance from zero is less than 3."

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Additional Examples

Example 3

A pediatrician recommends that a baby's bath water be 95°F, but it is acceptable for the temperature to vary from this amount by as much as 3°F. Write and solve an absolute-value inequality to find the range of acceptable temperatures. Graph the solutions.

$|t - 95| \leq 3$; $92 \leq t \leq 98$

Example 4

Solve each inequality.

A. $|x + 4| - 5 > -8$ all real numbers

B. $|x - 2| + 9 < 7$ no solutions

Also available online

Questioning Strategies

EXAMPLE 3

- How can you use an inequality to represent the phrase "at most"?

EXAMPLE 4

- How can you tell when an absolute-value inequality will have all real numbers as its solutions?
- How can you tell when an absolute-value inequality will have no solutions?

EXAMPLE 3 ***Safety Application***

Some fire extinguishers contain pressurized water. The water pressure should be 162.5 psi (pounds per square inch), but it is acceptable for the pressure to differ from this value by at most 12.5 psi. Write and solve an absolute-value inequality to find the range of acceptable pressures. Graph the solutions.

Let p represent the actual water pressure of a fire extinguisher.

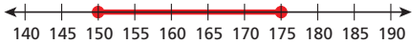
The difference between p and the ideal pressure	is at most	12.5 psi.
$\lvert p - 162.5 \rvert$	$\leq$	12.5

$|p - 162.5| \leq 12.5$

$p - 162.5 \geq -12.5$ AND $p - 162.5 \leq 12.5$ *Solve the two inequalities.*

$+ 162.5 \quad + 162.5 \qquad + 162.5 \quad + 162.5$

$p \geq 150$ AND $p \leq 175$



The range of acceptable pressures is $150 \leq p \leq 175$.

CHECK IT OUT!

3. A dry-chemical fire extinguisher should be pressurized to 125 psi, but it is acceptable for the pressure to differ from this value by at most 75 psi. Write and solve an absolute-value inequality to find the range of acceptable pressures. Graph the solutions.

3. $|p - 125| \leq 75$; $50 \leq p \leq 200$

When solving an absolute-value inequality, you may get a statement that is true for all values of the variable. In this case, all real numbers are solutions of the original inequality. If you get a false statement when solving an absolute-value inequality, the original inequality has no solutions.

EXAMPLE 4 ***Special Cases of Absolute-Value Inequalities***

Solve each inequality.

A $|x - 6| + 7 > 2$

$|x - 6| + 7 > 2$

$-7 \quad -7$ *Subtract 7 from both sides.*

$|x - 6| > -5$ *Absolute-value expressions are always nonnegative. Therefore, the statement is true for all values of x.*

All real numbers are solutions.

B $|x + 12| - 5 \leq -6$

$|x + 12| - 5 \leq -6$

$+5 \quad +5$ *Add 5 to both sides.*

$|x + 12| \leq -1$ *Absolute-value expressions are always nonnegative. Therefore, the statement is false for all values of x.*

The inequality has no solutions.

Remember!

An absolute value represents a distance, and distance cannot be less than 0.

CHECK IT OUT!

Solve each inequality.

4a. $|x| - 9 \geq -11$ all real numbers

4b. $4|x - 3.5| \leq -8$ no solutions

3 Close

Summarize

Solve each absolute-value inequality.

1. $4 + |x| < 16$ $-12 < x < 12$
2. $|x - 3| + 5 \leq -8$ no solutions
3. $|x + 1| - 4 \geq 3$ $x \leq -8$ OR $x \geq 6$
4. $2|x + 6| > -10$ all real numbers

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

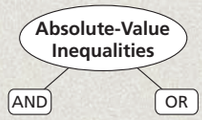
***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Describe how the solutions of $7|x| \le 21$ are different from the solutions of $7|x| < 21$.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the indicated type of absolute-value inequality and then solve.



Know it! Note

2-7 Exercises

Learn It Online
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Parent Resources Online

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1

1. $|x| - 5 \le -2$
2. $|x + 1| - 7.8 < 6.2$
3. $|3x| + 2 < 8$
4. $4|x| \le 20$
5. $|x - 5| + 1 < 2$
6. $\left|x + \frac{1}{2}\right| - \frac{1}{2} \le 3\frac{1}{2}$

SEE EXAMPLE 2

7. $|x| - 6 > 16$
8. $|x| + 2.9 > 8.6$
9. $2|x| \ge 8$
10. $|x + 2| > 7$
11. $|x - 3| + 2 \ge 4$
12. $|x + 5| - 4\frac{1}{2} \ge 7\frac{1}{2}$

SEE EXAMPLE 3

13. **Nutrition** A nutritionist recommends that an adult male consume 55 grams of fat per day. It is acceptable for the fat intake to differ from this amount by at most 25 grams. Write and solve an absolute-value inequality to find the range of fat intake that is acceptable. Graph the solutions.

SEE EXAMPLE 4

Solve each inequality.

14. $|x| + 8 \le 2$ no solutions
15. $|x + 3| < -5$ no solutions
16. $|x + 4| \ge -8$ all real numbers
17. $|x - 5| + \frac{1}{3} > -1$ all real numbers
18. $|3x| + 7 > 2$ all real numbers
19. $|x - 7| + 3.5 \le 2$ no solutions

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–25	1
26–31	2
32	3
33–38	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each inequality and graph the solutions.

20. $|x| + 6 \le 10$
21. $|x - 3| < 1$
22. $|x - 2| - 8 \le -3$
23. $|5x| < 15$
24. $|x - 2.4| + 4 \le 6.4$
25. $4 + |x + 3| < 7$
26. $|x - 1| > 2$
27. $6|x| \ge 60$
28. $|x - 4| + 3 > 8$
29. $2|x + 2| \ge 16$
30. $3 + |x - 4| > 4$
31. $\left|x - \frac{1}{2}\right| + 9 > 10\frac{1}{2}$
32. The thermostat for a sauna is set to 175 °F, but the actual temperature of the sauna may vary by as much as 12 °F. Write and solve an absolute-value inequality to find the range of possible temperatures. Graph the solutions.

Solve each inequality.

33. $12 + |x| \le 10$ no solutions
34. $\left|x + \frac{3}{5}\right| - 2 > -4$ all real numbers
35. $|x + 1| + 5 \ge 4$ all real numbers
36. $|4x| - 3 < -6$ no solutions
37. $3|x - 4| \le -9$ no solutions
38. $|2x| + 9 \ge 9$

Answers to *Think and Discuss*

1. Possible answer: The solutions of $7|x| \le 21$ are all values of x between -3 and 3, inclusive. The solutions of $7|x| < 21$ are all values of x between -3 and 3, not including these values.
2. See Additional Answers.

2-7 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**

Basic	20–31, 45–48
Average	20–31, 39, 40, 45–48
Advanced	20–31, 39, 40, 45–48, 58, 59

If you finished Examples **1–4**

Basic	20–38, 42–52, 55–57
Average	20–44, 46–52 even, 53–58
Advanced	20–44 even, 45–60

Homework Quick Check
Quickly check key concepts.
Exercises: 8, 10, 11, 12, 14, 24

Answers

1. $-3 \le x \le 3$

2. $-15 < x < 13$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 13, 32, 49–52

Reason abstractly and quantitatively. Exercises 39–41

Construct viable arguments and critique the reasoning of others. Exercises 53–54, 60

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State Resources Online

3. $-2 < x < 2$

4. $-5 \le x \le 5$

5. $4 < x < 6$

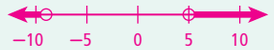
6. $-4\frac{1}{2} \le x \le 3\frac{1}{2}$

7. $x < -22$ OR $x > 22$

8. $x < -5.7$ OR $x > 5.7$

9. $x \le -4$ OR $x \ge 4$

10. $x < -9$ OR $x > 5$



11. $x \le 1$ OR $x \ge 5$

12. $x \le -17$ OR $x \ge 7$

13. $|x - 55| \le 25$; $30 \le x \le 80$

20–32. See Additional Answers.

MULTI-STEP TEST PREP **Exercise 52** involves using absolute-value inequalities to determine the range of possible prices for a CD. This exercise prepares students for the Multi-Step Test Prep.

Answers

39. Absolute-value expressions are always nonneg.

40. Absolute-value expressions are always nonneg.

41. Possible answer: $|x + 1| > -5$ has all real numbers as solutions, but $|x - 7| < 0$ has no solutions.

42. $|x| \leq 15$; $-15 \leq x \leq 15$

43. $|x - 2| \leq 3$; $-1 \leq x \leq 5$

44. $|x - 8| \geq 2$; $x \leq 6$ OR $x \geq 10$

45. $|a| \leq 2$

46. $|b| > 3$

47. $|c| \geq 6\frac{1}{2}$

48. $|d| < 7$

49. a. 10,010 Hz

    b. $|x - 10{,}010| \leq 9990$

50. European eel: $|x - 23| \leq 11$; Rainbow trout: $|x - 15| \leq 11$; Sea bass: $|x - 21| \leq 9$

Tell whether each statement is sometimes, always, or never true. Explain.

39. The value of $|x + 1|$ is greater than -5. **always**

40. The value of $|x - 7|$ is less than 0. **never**

41. An absolute-value inequality has all real numbers as solutions. **sometimes**

Write and solve an absolute-value inequality for each expression. Graph the solutions on a number line.

42. All numbers whose absolute value is less than or equal to 15

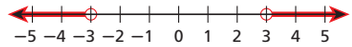
43. All numbers less than or equal to 3 units from 2 on the number line

44. All numbers at least 2 units from 8 on the number line

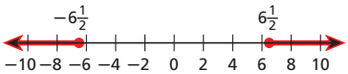
Write an absolute-value inequality for each graph.

45.

46.



47.



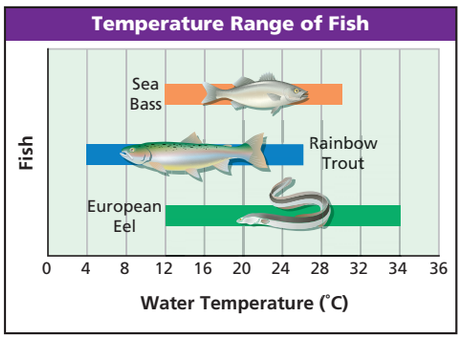
48.

49. Multi-Step The frequency of a sound wave determines its pitch. The human ear can detect a wide range of frequencies, from 20 Hz (very low notes) to 20,000 Hz (very high notes).

a. What frequency is at the middle of the range?

b. Write an absolute-value inequality for the range of frequencies the human ear can detect.

50. Biology The diagram shows the temperature range at which several fish species can survive. For each species, write an absolute-value inequality that gives the range of temperatures at which it can survive.



51. Entertainment On a game show, a contestant must guess a secret two-digit number. The secret number is 23. Write an absolute-value inequality that shows that the contestant's guess is more than 12 numbers away from the secret number.

$|n - 23| > 12$

52. The manager of a band recommends that the band sell its CDs for $8.75. The band decides to sell the CDs for p dollars.

a. Write an absolute-value expression that tells how far the band's price is from the recommended price. $|p - 8.75|$

b. The band wants the price of its CD to be no more than $1.25 from the recommended price. Write an absolute-value inequality that gives the range of possible prices for the CD. $|p - 8.75| \leq 1.25$

c. Solve the inequality. Write the solution as a compound inequality.

$\$7.50 \leq p \leq \$10.00$

(tl), © Brand X Pictures;

PRACTICE A

PRACTICE C

PRACTICE B

Solve each inequality and graph the solutions.

1. $|x| - 2 \leq 3$
   $x \geq -5$ AND $x \leq 5$
2. $|x + 1| + 5 < 7$
   $x > -3$ AND $x < 1$
3. $3|x - 6| \leq 9$
   $x \geq 3$ AND $x \leq 9$
4. $|x + 3| - 1.5 < -2.5$
   $x > -7$ AND $x < 1$
5. $|x| + 17 > 20$
   $x < -3$ OR $x > 3$
6. $|x - 6| - 7 > -3$
   $x < 2$ OR $x > 10$
7. $\frac{1}{2}|x + 5| \geq 2$
   $x \leq -9$ OR $x \geq -1$
8. $2|x - 2| \geq 3$
   $x \leq 0.5$ OR $x \geq 3.5$
9. The organizers of a drama club wanted to sell 350 tickets to their show. The actual sales were no more than 35 tickets from this goal. Write and solve an absolute-value inequality to find the range of the number of tickets that may have been sold.
   $|x - 350| \leq 35$; $315 \leq x \leq 385$
10. The temperature at noon in Los Angeles on a summer day was 88 °F. During the day, the temperature varied from this by as much as 7.5 °F. Write and solve an absolute-value inequality to find the range of possible temperatures for that day.
    $|x - 88| \leq 7.5$; $80.5 \leq x \leq 95.5$

READING STRATEGIES

You can use a number line to help you solve absolute-value inequalities that have the form $|x - b| \leq c$ or $|x - b| < c$. The following steps show how to use this method to solve $|x - 1| \leq 3$.

Step 1: Plot the value of b on the number line.
In $|x - 1| \leq 3$, $b = 1$

Step 2: Plot the two points that are c units away from b.
In this case, $c = 3$.

Step 3: Shade the points in between.
The solution is $-2 \leq x \leq 4$

Answer each question.

1. When you use a number line to solve the inequality $|x - 5| < 4$, which point should you plot first? 5
2. Which points should you plot next? 1 and 9
3. Graph the solutions of $|x - 5| < 4$ on the number line.
4. When you use a number line to solve the inequality $|x + 1| \leq 2$, which point should you plot first? *(Hint:* The inequality can be written as $|x - (-1)| \leq 2$) −1
5. Which points should you plot next? −3 and 1
6. Graph the solutions of $|x + 1| \leq 2$ on the number line.
7. Use this method to graph the solutions of $|x + 2| < 1$.

RETEACH

To solve an absolute-value inequality, first use inverse operations to isolate the absolute-value expression. Then write and solve a compound inequality.

Solve $|x - 2| + 8 < 10$.

Step 1: Isolate the absolute-value expression.

$|x - 2| + 8 < 10$

$-8 \quad -8$ *Subtract 8 from both sides.*

$|x - 2| < 2$

Step 2: Solve a compound inequality.

$|x - 2| < 2$ means $x - 2 > -2$ AND $x - 2 < 2$.

$+2 \quad +2 \qquad +2 \quad +2$ *Solve each inequality.*

$x > 0$ AND $x < 4$

Graph the solution as shown.

Solve each inequality and graph the solutions.

1. $|x| + 12 < 16$
   $x > -4$ AND $x < 4$
2. $|x - 1| + 5 \leq 9$
   $x \geq -3$ AND $x \leq 5$
3. $7|x| \leq 21$
   $x \geq -3$ AND $x \leq 3$
4. $|x + 4| - 3 < -2$
   $x > -5$ AND $x < -3$

53. Critical Thinking For which values of k does the inequality $|x| + 1 < k$ have no solutions? Explain.

54. Write About It Describe how to use an absolute-value inequality to find all the values on a number line that are within 5 units of −6.

TEST PREP

55. What is the solution of the inequality $3 + |x + 4| < 6$?

(A) $-13 < x < 5$ (C) $-6 < x < -2$

(B) $-7 < x < -1$ (D) $1 < x < 7$

56. A thermometer gives temperature readings that may be inaccurate by at most 2 °F. The actual temperature is 75 °F. Which absolute-value inequality describes the range of temperatures that may be shown on the thermometer?

(F) $|x - 75| \le 2$ (G) $|x + 75| \le 2$ (H) $|x - 75| \ge 2$ (J) $|x + 75| \ge 2$

57. The inequality $|w - 156| \le 3$ describes the weights of members of a wrestling team. Which statement is NOT true?

(A) All of the team members weigh no more than 159 pounds.

(B) A team member may weigh 152 pounds.

(C) Every member of the team is at most 3 pounds away from 156 pounds.

(D) There are no team members who weigh 160 pounds.

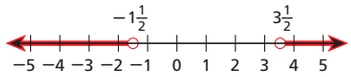
CHALLENGE AND EXTEND

Write an absolute-value inequality for each graph.

58.

58. Possible answer: $|x - 0.9| \le 3.3$

59.



59. Possible answer: $|x - 1| > 2\frac{1}{2}$

60. Critical Thinking Fill in the missing reasons to justify each step in solving $|2x - 6| + 5 \le 7$.

Statements	Reasons
1. $\|2x - 6\| + 5 \le 7$	Given
2. $\|2x - 6\| \le 2$	Subtr. Prop. of Ineq.
3. $2x - 6 \ge -2$ AND $2x - 6 \le 2$	Definition of absolute value
4. $2x \ge 4$ AND $2x \le 8$	Add. Prop. of Ineq.
5. $x \ge 2$ AND $x \le 4$	Div. Prop. of Ineq.

PROBLEM SOLVING

Write the correct answer.

1. A carpenter cuts boards that are 2 meters long. It is acceptable for the length to differ from this value by at most 0.05 meters. Write and solve an absolute-value inequality to find the range of acceptable lengths.
   $|x - 2| \le 0.05$; $1.95 \le x \le 2.05$
2. During a workout, Vince tries to keep his heart rate at 134 beats per minute. His actual heart rate varies from this value by as much as 8 beats per minute. Write and solve an absolute-value inequality to find Vince's range of heart rates.
   $|x - 134| \le 8$; $126 \le x \le 142$
3. Mai thinks of a secret number. She says that her secret number is more than 11 units away from 50. Write an absolute-value inequality that gives the possible values of Mai's number.
   $|x - 50| > 11$
4. Boxes of cereal are supposed to weigh 15.3 ounces each. A quality-control manager finds that the boxes are no more than 0.4 ounces away from this weight. Write an absolute-value inequality that gives the range of possible weights of the boxes.
   $|x - 15.3| \le 0.4$

The table gives the typical lifespan for several mammals. Use the table for questions 5-7. Select the best answer.

5. Which absolute-value inequality gives the number of years a goat may live?
   A $|x - 6| \le 11$ C $|x - 24| \le 6$
   B $|x - 15| \le 9$ D $|x - 30| \le 9$
6. Which mammal has a lifespan that can be represented by the absolute-value inequality $|x - 12.5| \le 2.5$?
   F Antelope H Otter
   G Koala J Wolf
7. The inequality $|x - 17| \le c$ gives the number of years a panda may live. What is the value of c?
   A 3 C 14
   B 6 D 20

Mammal	Lifespan (years)	Mammal	Lifespan (years)
Antelope	10 to 25	Otter	15 to 20
Goat	6 to 24	Panda	14 to 20
Koala	10 to 15	Wolf	13 to 15

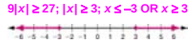
Source: http://www.sandiegozoo.org/animalbytes/a-mammal.html

CHALLENGE

When you solve a multi-step inequality, you may need to simplify expressions, combine like terms, or use the Distributive Property. This also applies when you solve inequalities that involve absolute values.

For example, to solve $3(|x| + 4) < 18$, you can first use the Distributive Property on the left side of the equation to write $3|x| + 12 < 18$.

1. Show how to finish solving the above inequality. Graph the solutions.
   $3|x| < 6$; $|x| < 2$; $x > -2$ AND $x < 2$



In some cases, you may need to combine like terms. For example, to solve $6|x| - 5 + 3|x| \ge 22$, first combine the absolute-value terms to write $9|x| - 5 \ge 22$.

2. Show how to finish solving the above inequality. Graph the solutions.
   $9|x| \ge 27$; $|x| \ge 3$; $x \le -3$ OR $x \ge 3$

Solve each inequality and graph the solutions.

3. $5|x| + 1 - 3|x| \le 3$
   $x \ge -1$ AND $x \le 1$

4. $2(5 + |x|) < 13$
   $x > -\frac{3}{2}$ AND $x < \frac{3}{2}$

5. $7|x + 1| - 5|x + 1| > 4$
   $x < -3$ OR $x > 1$

6. $3(|x - 2| + 1) \ge -1$
   all real numbers

COMMON ERROR ALERT

When writing an absolute-value inequality of the form $|x - k| \le a$, students sometimes confuse the values that should be used for k and a. In **Exercise 43**, students may write $|x - 3| \le 2$. Explain that the value that is at the center of the range (in this case, 2) should appear inside the absolute-value expression, so the correct inequality is $|x - 2| \le 3$.

TEST PREP DOCTOR If students answered **H** or **J** for **Exercise 56**, remind them that the range of values is represented by AND, which corresponds to a less-than symbol. Therefore, choices **H** and **J** can be eliminated at once.

Answers

53, 54 See Additional Answers.

Journal

Have students compare the steps they would use to solve $|x + 1| - 5 < 4$ with the steps they would use to solve $|x + 1| - 5 > 4$.

ALTERNATIVE ASSESSMENT

Have students write and solve an absolute-value inequality that involves AND and an absolute-value inequality that involves OR.

Power Presentations with PowerPoint®

2-7 Lesson Quiz

Solve each inequality and graph the solutions.

1. $3|x| > 15$ $x < -5$ OR $x > 5$

2. $|x + 3| + 1 < 3$ $-5 < x < -1$

3. A number n is no more than 7 units away from 5. Write and solve an absolute-value inequality to show the range of possible values for n.
   $|n - 5| \le 7$; $-2 \le n \le 12$

Solve each inequality.

4. $|3x| + 1 < 1$ no solutions
5. $|x + 2| - 3 \ge -6$ all real numbers

Also available online

SECTION 2B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and and skills in a real-world format.

INTERACTIVE **Online Edition**

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

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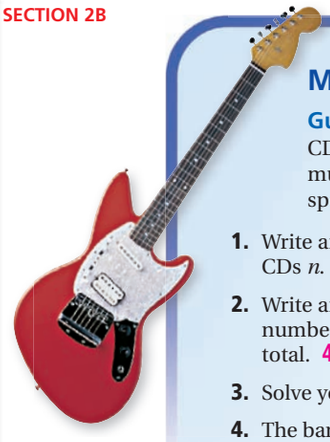
CHAPTER 2

SECTION 2B

MULTI-STEP TEST PREP



MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Multi-Step and Compound Inequalities

Guitar Picks Cullen and his band are interested in recording a CD of their music. The recording studio charges $450 to record the music and then charges $5 for each CD. The band is required to spend at least $1000 for the total of the recording and CD charges.

1. Write an equation for the cost c of the CDs based on the number of CDs n. $c = 450 + 5n$
2. Write an inequality that can be used to determine the minimum number of CDs that must be burned at this studio to meet the $1000 total. $450 + 5n \geq 1000$
3. Solve your inequality from Problem 2. $n \geq 110$ where n is a natural number
4. The band orders the minimum number of CDs found in Problem 3. They want to sell the CDs and make at least as much money as they spent for the recording studio and making the CDs. Write an inequality that can be solved to determine the minimum amount the band should charge for their CDs. $110x \geq 1000$
5. Solve your inequality from Problem 4. $x \geq 9.09$
6. If the band has 30 more CDs made than the minimum number found in Problem 3 and charges the minimum price found in Problem 5, will they make a profit? If so, how much profit will the band make? yes; $122.60

(t), ©Brand X Pictures; (br), Peter Beavis/Taxi/Getty Images; (cr), PhotoDisc/Getty Images; (b), PhotoDisc/Getty Images

INTERVENTION

Scaffolding Questions

1. What is the cost if Cullen's band purchases only 1 CD? $455 What is the cost if the band purchases 10 CDs? $500
2–3. What inequality symbol is indicated by the word *minimum*? greater than or equal to Would a graph of the inequality be shaded or have individual points? Explain. individual points, because they must buy whole numbers of CDs
4–5. What expression can be used to represent the amount made from selling the CDs? $110x$ Would the endpoint on a graph of the inequality have an empty or a solid circle? solid
6. How is the profit from the sale of CDs determined? sales minus cost

Extension

Write and solve an inequality to find the number of CDs they would need to purchase and sell for $9 each in order to make at least $200 profit. $200 \leq 9x - (450 + 5x)$; $x \geq 162.5$; at least 163 CDs

Quiz for Lessons 2-4 Through 2-7

2-4 Solving Two-Step and Multi-Step Inequalities

Solve each inequality and graph the solutions.

1. $2x + 3 < 9$ $x < 3$
2. $3t - 2 > 10$ $t > 4$
3. $7 \geq 1 - 6r$ $r \geq -1$

Solve each inequality.

4. $2(x - 3) > -1$ $x > 2.5$
5. $\frac{1}{3}a + \frac{1}{2} > \frac{2}{3}$ $a > \frac{1}{2}$
6. $15 < 5(m - 7)$ $m > 10$
7. $2 + (-6) > 0.8p$ $p < -5$
8. The average of Mindy's two test scores must be at least 92 to make an A in the class. Mindy got an 88 on her first test. What scores can she get on her second test to make an A in the class? Mindy must make a score of 96 or higher.

2-5 Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.

9. $5x < 3x + 8$ $x < 4$
10. $6p - 3 > 9p$ $p < -1$
11. $r - 8 \geq 3r - 12$ $r \leq 2$

Solve each inequality.

12. $3(y + 6) > 2(y + 4)$ $y > -10$
13. $4(5 - g) \geq g$ $g \leq 4$
14. $4x < 4(x - 1)$ no solutions
15. $3(1 - x) \geq -3(x + 2)$ all real numbers
16. Phillip has \$100 in the bank and deposits \$18 per month. Gil has \$145 in the bank and deposits \$15 per month. For how many months will Gil have a larger bank balance than Phillip? 15 mo

2-6 Solving Compound Inequalities

Solve each compound inequality and graph the solutions.

17. $-2 \leq x + 3 < 9$ $-5 \leq x < 6$
18. $m + 2 < -1$ OR $m - 2 > 6$ $m < -3$ OR $m > 8$
19. $-3 \geq x - 1 > 2$ no solutions
20. $-2 > r + 2$ OR $r + 4 < 5$ $r < 1$
21. It is recommended that a certain medicine be stored in temperatures above 32 °F and below 70 °F. Write a compound inequality to show the acceptable storage temperatures for this medicine. $32 < t < 70$

2-7 Solving Absolute-Value Inequalities

Solve each inequality and graph the solutions.

22. $|x| + 9 \leq 12$ $-3 \leq x \leq 3$
23. $|x + 7| - 15 < 6$ $-28 < x < 14$
24. $4.5|x| \geq 31.5$ $x \leq -7$ OR $x \geq 7$

Solve each inequality.

25. $|x - 2| \leq 14$ $-12 \leq x \leq 16$
26. $|x| - 9.2 < -5.7$ $-3.5 < x < 3.5$
27. $\frac{1}{2} + 2|x| > -4$ all real numbers
28. $7 + |3x| > 13$ $x < -2$ OR $x > 2$
29. Eli attended a concert. The decibel level of the music averaged 110 decibels but varied by 22 decibels from the average. Write and solve an absolute-value inequality to find the decibel range. Graph the solutions. $|d - 110| \leq 22$; $88 \leq d \leq 132$

READY TO GO ON? SECTION 2B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

1–3, 9–11, 17–20, 22–24, 29. For graphs, see Additional Answers.

READY TO GO ON?
Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 2B		
	Worksheets	Online	
Lesson 2-4	2-4 Intervention	Activity 2-4	Diagnose and Prescribe Online
Lesson 2-5	2-5 Intervention	Activity 2-5	
Lesson 2-6	2-6 Intervention	Activity 2-6	
Lesson 2-7	2-7 Intervention	Activity 2-7	

***Ready to Go On? Enrichment*, Section 2B**
Worksheets
Online

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

Countdown

Resources

Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

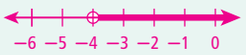
Answers

1. inequality
2. union
3. compound inequality
4. intersection
5. solution of an inequality
6.

7.

8.

9.

10. 

11.

12. $a < 2$
13. $k \geq -3.5$
14. $q < -10$
15. t = temperature; $t \geq 72$

16. s = students; $s \leq 12$ where s is a natural number

Vocabulary

compound inequality	**intersection**	**union**
inequality	**solution of an inequality**	

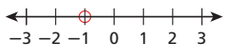
Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a mathematical statement that two quantities are not equal.
2. The numbers that are solutions to either inequality of a compound inequality is the ___?___.
3. A statement formed by combining two simple inequalities with the words AND or OR is a(n) ___?___.
4. The numbers that are solutions to both inequalities of a compound inequality is the ___?___.
5. Any value that makes the inequality true is a(n) ___?___.

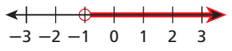
2-1 Graphing and Writing Inequalities

EXAMPLES

■ **Graph the inequality $y > -1$.**

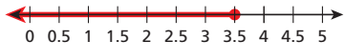


Draw an empty circle at −1.



Shade all the numbers greater than −1.

■ **Write the inequality shown by the graph.**

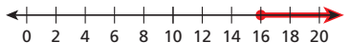


$n \leq 3.5$ *Use the variable n. The arrow points left, so use either < or ≤. The solid circle means 3.5 is a solution, so use ≤.*

■ **Write an inequality for the situation and graph the solutions.**

Applicants for a driver's permit must be at least 16 years old.

age	must be at least	16 years
a	$\geq$	16



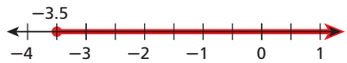
EXERCISES

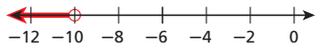
Graph each inequality.

6. $x > -3$
7. $p \leq 4$
8. $-1 > t$
9. $r \geq 9.5$
10. $2(3 - 5) < k$
11. $w < 3$

Write the inequality shown by each graph.

12.

13. 

14. 

Define a variable and write an inequality for each situation. Graph the solutions.

15. The temperature must be at least 72 °F.
16. No more than 12 students were present.
17. It takes less than 30 minutes to complete the lab activity.

17. m = minutes; $m < 30$ where m is positive

2-2 Solving Inequalities by Adding or Subtracting

EXAMPLES

Solve each inequality and graph the solutions.

■ $x + 6 > 2$

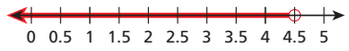
$x + 6 > 2$
$-6 \quad -6$
$x > -4$

Since 6 is added to x, subtract 6 from both sides.

■ $n - 1.3 < 3.2$

$n - 1.3 < 3.2$
$+1.3 \quad +1.3$
$n < 4.5$

Since 1.3 is subtracted from x, add 1.3 to both sides.



EXERCISES

Solve each inequality and graph the solutions.

18. $t + 3 < 10$

19. $k - 7 \leq -5$

20. $-1 < m + 4$

21. $x + 2.3 \geq 6.8$

22. $w - 3 < 6.5$

23. $4 > a - 1$

24. $h - \frac{1}{4} < \frac{3}{4}$

25. $5 > 7 + v$

26. Tammy wants to run at least 10 miles per week. So far this week, she ran 4.5 miles. Write and solve an inequality to determine how many more miles Tammy must run this week to reach her goal.

27. Rob has a gift card for $50. So far, he has selected a shirt that costs $32. Write and solve an inequality to determine the additional amount Rob could spend without exceeding the gift card limit.

2-3 Solving Inequalities by Multiplying or Dividing

EXAMPLES

■ **Solve $\frac{p}{-3} \leq 6$ and graph the solutions.**

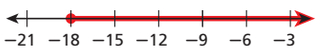
$\frac{p}{-3} \leq 6$

Since p is divided by −3, multiply both sides by −3.

$-3 \cdot \frac{p}{-3} \geq -3 \cdot 6$

$p \geq -18$

Change ≤ to ≥.



■ **Pizzas cost $5.50 each. What are the possible numbers of pizzas that can be purchased with $30?**

Let n represent the number of pizzas that can be purchased.

$5.50	times	number of pizzas	is at most	$30.
5.50	•	n	≤	30

$5.50n \leq 30$

$\frac{5.50n}{5.50} \leq \frac{30}{5.50}$

Since n is multiplied by 5.50, divide both sides by 5.50.

$n \leq 5\frac{5}{11}$

Only a whole number of pizzas can be purchased, so 0, 1, 2, 3, 4, or 5 pizzas can be purchased.

EXERCISES

Solve each inequality and graph the solutions.

28. $3a \leq 15$

29. $-18 < 6t$

30. $\frac{p}{4} > 2$

31. $\frac{2}{5}x \leq -10$

32. $-3n < -18$

33. $\frac{g}{-2} > 6$

34. $-2k < 14$

35. $-3 > \frac{1}{3}r$

36. $27 < -9h$

37. $-0.4g > -1$

38. Notebooks cost $1.39 each. What are the possible numbers of notebooks that can be purchased with $10?

39. A senior class is selling lanyards as a fundraiser. The profit for each lanyard is $0.75. Write and solve an inequality to determine the number of lanyards the class must sell to make a profit of at least $250.

36. $h < -3$

37. $g < 2.5$

38. 0, 1, 2, 3, 4, 5, 6, 7

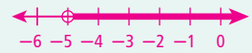
39. $0.75n \geq 250$; $n \geq 333\frac{1}{3}$; they must sell at least 334 lanyards.

Answers

18. $t < 7$

19. $k \leq 2$

20. $m > -5$



21. $x \geq 4.5$

22. $w < 9.5$

23. $a < 5$

24. $h < 1$

25. $v < -2$

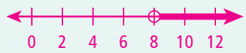
26. $4.5 + m \geq 10$; $m \geq 5.5$; Tammy must run 5.5 mi or more.

27. $32 + d \leq 50$; $d \leq 18$; Rob can spend $18 or less.

28. $a \leq 5$

29. $t > -3$

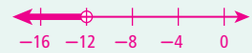
30. $p > 8$



31. $x \leq -25$

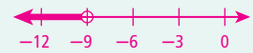
32. $n > 6$

33. $g < -12$



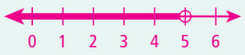
34. $k > -7$

35. $r < -9$



Answers

40. $x < 5$



41. $t \geq 6$

42. $m > -11$

43. $x < -1$

44. $h > -3$

45. $x > 1\frac{1}{2}$

46. $b \leq 10$

47. $y > 3\frac{1}{2}$

48. $n > -15$

49. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, or 13

50. $m < -1$

51. $y \geq -2$

52. $c < -3$

53. $q \leq -4$

54. $x > 2$

55. $t < 3$

56. no solutions

57. all real numbers

58. $p > -\frac{1}{2}$

2-4 Solving Two-Step and Multi-Step Inequalities

EXAMPLES

Solve each inequality and graph the solutions.

■ $18 + 3t > -12$

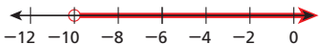
$18 + 3t > -12$ — *Since 18 is added to 3t, subtract 18 from both sides.*

$-18 \quad -18$

$3t > -30$

$\frac{3t}{3} > \frac{-30}{3}$ — *Since t is multiplied by 3, divide both sides by 3.*

$t > -10$



■ $3^2 - 5 \leq 2(1 + x)$

$3^2 - 5 \leq 2(1 + x)$ — *Simplify the left side using order of operations.*

$9 - 5 \leq 2(1 + x)$

$4 \leq 2(1 + x)$ — *Distribute 2 on the right side.*

$4 \leq 2(1) + 2(x)$

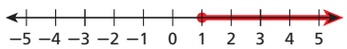
$4 \leq 2 + 2x$ — *Since 2 is added to 2x, subtract 2 from both sides*

$-2 \quad -2$

$2 \leq 2x$

$\frac{2}{2} \leq \frac{2x}{2}$ — *Since x is multiplied by 2, divide both sides by 2.*

$1 \leq x$



EXERCISES

Solve each inequality and graph the solutions.

40. $3x + 4 < 19$

41. $7 \leq 2t - 5$

42. $\frac{m + 3}{2} > -4$

43. $2(x + 5) < 8$

44. $-4(2 - 5) > (-3)^2 - h$

45. $\frac{1}{5}x + \frac{1}{2} > \frac{4}{5}$

46. $0.5(b - 2) \leq 4$

47. $\frac{1}{3}y - \frac{1}{2} > \frac{2}{3}$

48. $6 - 0.2n < 9$

49. Carl's Cable Company charges \$55 for monthly service plus \$4 for each pay-per-view movie. Teleview Cable Company charges \$110 per month with no fee for movies. For what number of movies is the cost of Carl's Cable Company less than the cost of Teleview?

2-5 Solving Inequalities with Variables on Both Sides

EXAMPLES

■ **Solve $b + 16 < 3b$ and graph the solutions.**

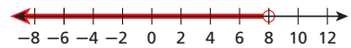
$b + 16 > 3b$ — *Subtract b from both sides so that the coefficient of b is positive.*

$-b \quad -b$

$16 > 2b$

$\frac{16}{2} > \frac{2b}{2}$ — *Since b is multiplied by 2, divide both sides by 2.*

$8 > b$



■ **Solve the inequality $3(1 + k) > 4 + 3k$.**

$3 + 3k > 4 + 3k$ — *Distribute 3 on the left side.*

The same variable term ($3k$) appears on both sides.

For any number $3k$, adding 3 will never result in a greater number than adding 4.

There are no solutions.

EXERCISES

Solve each inequality and graph the solutions.

50. $5 + 2m < -3m$

51. $y \leq 6 + 4y$

52. $4c - 7 > 9c + 8$

53. $-3(2 - q) \geq 6(q + 1)$

54. $2(5 - x) < 3x$

55. $3.5t - 1.8 < 1.6t + 3.9$

Solve each inequality.

56. $d - 2 < d - 4$

57. $2(1 - x) > -2(1 + x)$

58. $4(1 - p) < 4(2 + p)$

59. $3w + 1 > 3(w - 1)$

60. $5(4 - k) < 5k$

61. $3(c + 1) > 3c + 5$

62. Hanna has a savings account with a balance of \$210 and deposits \$16 per month. Faith has a savings account with a balance of \$175 and deposits \$20 per month. Write and solve an inequality to determine the number of months Hanna's account balance will be greater than Faith's account balance.

59. all real numbers

60. $k > 2$

61. no solutions

62. $210 + 16m > 175 + 20m$; $8.75 > m$

2-6 Solving Compound Inequalities

EXAMPLES

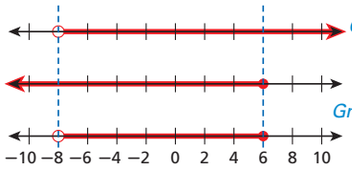
Solve each compound inequality and graph the solutions.

■ $-3 < c + 5 \leq 11$

$-5 \quad -5 \quad -5$

$-8 < c \quad \leq 6$

Since 5 is added to c, subtract 5 from each part of the inequality.



Graph $c > -8$ and $c \leq 6$.

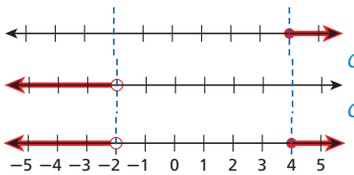
Graph the intersection.

■ $-2 + t \geq 2$ OR $t + 3 < 1$

$+2 \quad +2 \quad -3 \quad -3$

$t \geq 4$ OR $t < -2$

Solve the simple inequalities.



Graph $t \geq 4$ and $t < -2$.

Graph the union.

EXERCISES

Solve each compound inequality and graph the solutions.

63. $-4 < t + 6 < 10$

64. $-8 < k - 2 \leq 5$

65. $-3 + r > 4$ OR $r + 1 < -1$

66. $2 > n + 3 > 5$

67. $12 \geq p + 7 > 5$

68. $3 < s + 9$ OR $1 > s - 4$

69. One day, the high temperature was 84 °F and the low temperature was 68 °F. Write a compound inequality to represent the day's temperatures.

70. The table shows formulas for the recommended heart rates during exercise for a person who is a years old. Write and solve a compound inequality to determine the heart rate range for a 16-year-old person.

Recommended Heart Rate Range	
Lower Limit	$0.5 \times (220 - a)$
Upper Limit	$0.9 \times (220 - a)$

2-7 Solving Absolute-Value Inequalities

EXAMPLES

Solve each inequality and graph the solutions.

■ $|x| + 4 < 9$

$|x| + 4 < 9$

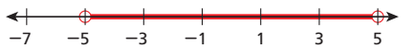
$-4 \quad -4$

$|x| \quad < 5$

Subtract 4 from both sides.

$x > -5$ AND $x < 5$

Write as a compound inequality.



■ $|x - 3| + 7 \geq 13$

$-7 \quad -7$

$|x - 3| \quad \geq 6$

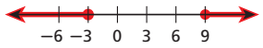
Subtract 7 from both sides.

$x - 3 \leq -6$ OR $x - 3 \geq 6$

$+3 \quad +3 \quad +3 \quad +3$

$x \quad \leq -3$ OR $x \quad \geq 9$

Solve the two inequalities.



EXERCISES

Solve each inequality and graph the solutions.

71. $|x| - 7 \leq 15$

72. $|x + 4| > 8$

73. $6|x| \leq 24$

74. $|x + 9| + 11 < 20$

75. $3|x| \geq 9$

76. $4|2x| < 24$

Solve the inequality.

77. $|x| - 5.4 > 8.5$

78. $|5.2 + x| < 7.3$

79. $|x - 7| + 10 \geq 12$

80. $14|x| - 15 \geq 41$

81. $\left|x - \frac{1}{2}\right| + 4 \leq \frac{5}{2}$

82. $|x + 5.5| - 6.4 \leq 4.9$

83. The water depth for a pool is set to 6 ft, but the actual depth of the pool may vary by as much as 4 in. Write and solve an absolute-value inequality to find the range of possible water depths in inches. Graph the solutions.

Answers

63. $-10 < t < 4$

64. $-6 < k \leq 7$

65. $r > 7$ OR $r < -2$

66. no solutions

67. $-2 < p \leq 5$

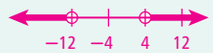
68. all real numbers

69. $68 \leq t \leq 84$

70. $102 \leq n \leq 183.6$

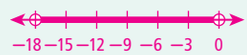
71. $-22 \leq x \leq 22$

72. $x < -12$ OR $x > 4$



73. $-4 \leq x \leq 4$

74. $-18 < x < 0$



75. $x \leq -3$ OR $x \geq 3$

76. $-3 < x < 3$

77. $x < -13.9$ OR $x > 13.9$

78. $-12.5 < x < 2.1$

79. $x \leq 5$ OR $x \geq 9$

80. $x \leq -4$ OR $x \geq 4$

81. no solutions

82. $-16.8 \leq x \leq 5.8$

83. $|d - 72| \leq 4$; $68 \leq d \leq 76$

CHAPTER 2 CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

Online Edition

Resources

Assessment Resources

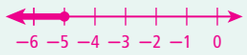
Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Answers

1. all real numbers greater than or equal to −6
2. all real numbers greater than 4
3. all real numbers less than or equal to −2
4. all real numbers less than or equal to 8
5.

6.

7. 

8.

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State Resources Online

CHAPTER 2 CHAPTER TEST

Describe the solutions of each inequality in words.

1. $-6 \le m$
2. $3t > 12$
3. $-x \ge 2$
4. $2 + b \le 10$

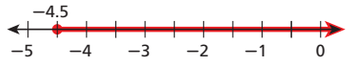
Graph each inequality.

5. $b > -3$
6. $2.5 < c$
7. $y \le -\sqrt{25}$
8. $3 - (4 + 7) \ge h$

Write the inequality shown by each graph.

9.

$d < 1$

10. 

$p \ge -4.5$

Write an inequality for the situation and graph the solutions.

11. Madison must run a mile in no more than 9 minutes to qualify for the race.
$m \le 9$ where m is nonnegative

Solve each inequality and graph the solutions.

12. $d - 5 > -7$ $d > -2$
13. $f + 4 < -3$ $f < -7$
14. $4.5 \ge s + 3.2$ $s \le 1.3$
15. $g + (-2) \le 9$ $g \le 11$

16. Students need at least 75 hours of volunteer service to meet their graduation requirement. Samir has already completed 48 hours. Write and solve an inequality to determine how many more hours he needs to complete.
$48 + h \ge 75$; $h \ge 27$; Samir needs at least 27 more hours.

Solve each inequality and graph the solutions.

17. $-2c \le 2$ $c \ge -1$
18. $3 > \frac{k}{2}$ $k < 6$
19. $\frac{4}{5}x \le -8$ $x \le -10$
20. $\frac{b}{3} > -7$ $b > -21$

21. Marco needs to buy premium gasoline for his car. He has $20 in his wallet. Write and solve an inequality to determine how many gallons of gas Marco can buy.
$2.25g \le 20$; $g \le 8.89$; Marco can buy 8.89 gallons or less.

Gasoline Prices ($)		
Regular	Plus	Premium
2.05	2.12	2.25

Solve each inequality and graph the solutions.

22. $3x - 8 < 4$ $x < 4$
23. $-2(c - 3) > 4$ $c < 1$
24. $5 \le \frac{3}{4}n - 2^4$ $n \ge 28$
25. $3 - 2a \le -15 + (-9)$ $a \ge 13\frac{1}{2}$

Solve each inequality.

26. $2k - 6 > 3k + 2$ $k < -8$
27. $2(5 - f) \le f + 12$ $f \ge -\frac{2}{3}$
28. $\frac{3}{2}d \le -\frac{1}{2}d + 6$ $d \le 3$

Solve each compound inequality and graph the solutions.

29. $-1 \le x - 3 < 3$ $2 \le x < 6$
30. $t + 7 < 3$ OR $t - 1 > 4$ $t < -4$ OR $t > 5$
31. $4 \le d - 2 < 5$ $6 \le d < 7$

32. The driving school instructor has asked Lina to stay within 2 miles of the posted speed limits. The current road has a speed limit of 45 mi/h. Write a compound inequality to show Lina's acceptable speeds s. $43 \le s \le 47$

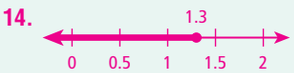
Solve each inequality.

33. $|x - 3| + 7 < 17$ $-7 < x < 13$
34. $6|x| + 4 \ge 16$ $x \le -2$ OR $x \ge 2$
35. $|x + 12| \le 23$ $-35 \le x \le 11$

11.

12.

13.

14. 

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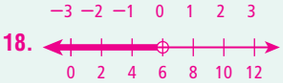
18. 

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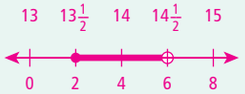
29. 

30.

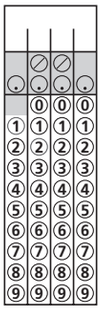
31.

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 2

FOCUS ON SAT STUDENT-PRODUCED RESPONSES

Ten questions on the SAT require you to enter your answer in a special grid like the one shown. You do not have to write your answer in the boxes at the top of the grid, but doing this may help you avoid errors when filling in the grid. The circles must be filled in correctly for you to receive credit.



HOT TIP!

You cannot enter a zero in the first column of the grid. This is to encourage you to give a more accurate answer when you need to round. For example, $\frac{1}{16}$ written as a decimal is 0.0625. This should be entered in the grid as .063 instead of 0.06.

You may want to time yourself as you take this practice test. It should take you about 9 minutes to complete.

1. Mailing a standard-sized letter in 2005 by first-class mail cost \$0.37 for a letter weighing 1 ounce or less and \$0.23 for each additional ounce. How much did it cost, in dollars, to send a standard-sized letter that weighed 3 ounces?
0.83

2. If $p = q - 2$ and $\frac{q}{3} = 9$, what is the value of p?
25

3. Give the maximum value of x if $12 - 3(x + 1) \geq \frac{1}{2}(3 - 5)$.
$\frac{10}{3}$

4. Give the minimum value of x if $2x + y \leq 7x - 9$ and $y = -3$.
$\frac{6}{5}$

5. For what integer value of x is $2x - 9 < 5$ and $x - 1 > 4$? **6**

6. What is the minimum value of z that satisfies the inequality $z - 7.3 \geq 4.1$?
11.4

7. To be eligible for financial aid, Alisa must work at least 15 hours per week in a work-study program. She wants to spend at least 5 more hours studying than working each week. What is the minimum number of hours per day (Monday through Friday) that she must study to meet this goal and be eligible for financial aid?
4

8. For all real numbers a and b, define the operation # as follows:

$$a \# b = 2a - b$$

Given $a = 3$ and $a \# b = 1$, what is the value of b?
5

Organizer

Objective: Provide practice for college entrance exams such as the SAT.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the SAT represent the following math content areas:

Number and Operations, 20–25%

Algebra and Functions, 35–40%

Geometry and Measurement, 25–30%

Data Analysis, Statistics, and Probability, 10–15%

Items on this page focus on:

- Number and Operations
- Algebra and Functions

TEST PREP DOCTOR

1. Students who answered 1.06 may have found the cost of 3 additional ounces instead of 2 additional ounces.

2. Students who answered 1 may have incorrectly solved for q and then substituted $q = 3$ into the first equation.

3. Students who gave an answer greater than $\frac{10}{3}$ may have forgotten to reverse the inequality symbol when multiplying or dividing by a negative number.

4. Students who gave an answer less than -24 may have substituted -3 for x instead of y. Urge students to read each test item carefully.

5. Students who gave an answer other than 6 that is between 5 and 7 may have solved correctly, but did not remember to give an integer answer.

6. Students who gave an answer of 3.2 may have subtracted instead of adding.

7. Students may have difficulty working with two inequalities. Suggest that students first write the inequality and then test several values to determine whether their answer is reasonable.

8. Students may have difficulty approaching the problem. Suggest that students write the given equation and then substitute 1 for the expression on the left side and 3 for a on the right side.

CHAPTER 2 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR This Test Tackler focuses on writing full-credit answers to short-response test items. Students benefit from knowing how their answers will be scored. By seeing examples of answers that do not earn full credit, students are reminded that they must pay attention to all parts of the question to receive full credit for an answer.

Learn It Online
State Resources Online

Short Response: Understand Short Response Scores

To answer a short-response question completely, you must show how you solved the problem and explain your answer. Short response questions are scored using a 2-point scoring rubric. A sample scoring rubric is provided below.

EXAMPLE 1

Short Response An online company offers free shipping if the cost of the order is at least $35. Your order currently totals $26.50. Write an inequality to show how much more you need to spend to qualify for free shipping. Solve the inequality and explain what your answer means.

2-point response:

Let c be the amount I must add to my order.
c plus the amount I already ordered must be at least $35.
$c + 26.50 \geq 35$
$c + 26.50 \geq 35$
$c + 26.50 - 26.50 \geq 35 - 26.50$
$c \geq 8.50$
Check:
$8.50 + 26.50 \geq 35$ ✓
To get free shipping on the order, I must spend at least $8.50 more since $8.50 + $26.50 is at least $35.

The student wrote and solved an inequality correctly. The student showed all work and explained the meaning of the solution to the inequality.

1-point response:

$c + 26.50 > 35$
$c > 8.50$
$8.50

The student gave a correct answer, but the inequality symbol shown in the student's work is incorrect. No explanation was given.

0-point response:

$9.25

The student gave an answer that satisfies the problem but did not show any work or give an explanation.

Scoring Rubric:

2 points: The student writes and correctly solves an inequality, showing all work. Student defines the variable, answers the question in a complete sentence, and provides an explanation.

1 point: The student writes and correctly solves an inequality but does not show all work, does not define the variable, or does not provide an explanation.

1 point: The student writes and solves an inequality but gives an incorrect answer. The student shows all work and provides an explanation for the answer.

0 points: The student gives no response or provides a solution without showing any work or explanation.

HOT TIP! Read short-response test items carefully. If you are allowed to write in the test booklet, underline or circle the parts of the question that tell you what your answer must include. Be sure to explain how you get your answer in complete sentences.

Read each sample and answer the questions that follow by using the scoring rubric below.

Scoring Rubric:

2 points: The student demonstrates a thorough understanding of the concept, correctly answers the question, and provides a complete explanation.

1 point: The student correctly answers the question but does not show all work or does not provide an explanation.

1 point: The student makes minor errors resulting in an incorrect solution but shows and explains understanding of the concept.

0 points: The student gives a response but shows no work or explanation, or the student gives no response.

Sample A

Short Response Write a real-world situation that can be modeled by the inequality $25s - 75 \geq 250$. Solve for s and explain how the value of s relates to your situation.

Student's Answer

A painter rents a booth at the county fair for $75. The artist sells his paintings for $25 each. If he makes at least $250 in profit, he can buy a new easel.

The artist has to sell at least 13 paintings.

1. What score should the student's answer receive? Explain your reasoning.
2. What additional information, if any, should the student's answer include in order to receive full credit?

Sample B

Short Response How do the solutions of $3s - 10 < 15 - 2s$ and $-34 + 9s \leq 4s - 9$ differ? How are the solutions alike? Include a graph in your explanation.

Student's Answer

Solve both inequalities.

$3s - 10 < 15 - 2s$

$+2s \quad +2s$

$5s - 10 < 15$

$+10 \quad +10$

$5s < 25$

$\frac{5s}{5} < \frac{25}{5}$

$s < 5$

$-34 + 9s \leq 4s - 9$

$-4s \quad -4s$

$-34 + 5s \leq -9$

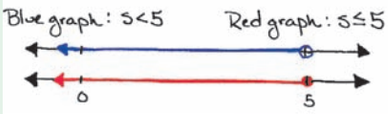
$+34 \quad +34$

$5s \leq 25$

$\frac{5s}{5} \leq \frac{25}{5}$

$s \leq 5$

Blue graph: $s < 5$ Red graph: $s \leq 5$



3. What score should the student's answer receive? Explain your reasoning.
4. What additional information, if any, should the student's answer include in order to receive full credit?

Sample C

Short Response Explain the difference between the solution of the equation $x - 6 = 2x + 9$ and the solutions of the inequality $x - 6 < 2x + 9$.

Student's Answer

The equation has a solution of x = –15, and the inequality has a solution of x > –15. The equation is true only when x equals –15. The inequality is true for all values greater than –15.

5. What score should the student's answer receive? Explain your reasoning.
6. What additional information, if any, should the student's answer include in order to receive full credit?

Answers

1. 1; the student did not show how he or she got the answer 13.
2. The student should explain how he or she got the answer by showing each step he or she followed to solve the inequality.
3. 1; the student did not explain how the graphs and solutions differ or how they are alike.
4. The student should explain that the solutions of the two inequalities are the same except that one includes 5 and the other does not.
5. 1; the student did not show the steps he or she followed to solve the equation or the inequality.
6. The student should show each step he or she followed to solve the equation and the inequality.

Answers to Test Items

A. See answer to Problem 2.

B. See answer to Problem 4.

C. See answer to Problem 6.

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources
Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

Learn It Online
State Test Practice

CUMULATIVE ASSESSMENT

Multiple Choice

1. Which algebraic expression means "5 less than y"?

Ⓐ $5 - y$
Ⓑ $y - 5$
Ⓒ $5 < y$
Ⓓ $5 \div y$

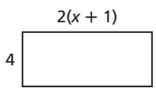
2. Which expression is equivalent to $5 + 2(x - 5)$?

Ⓕ $2x$
Ⓖ $2x + 5$
Ⓗ $2x - 5$
Ⓙ $7x - 35$

3. If $t + 8 = 2$, find the value of $2t$.

Ⓐ -12
Ⓑ -6
Ⓒ 12
Ⓓ 20

4. The length of the rectangle is $2(x + 1)$ meters and the perimeter is 60 meters. What is the length of the rectangle?

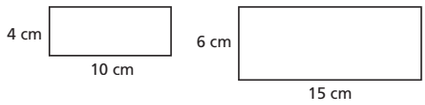


Ⓕ 12 meters
Ⓖ 26 meters
Ⓗ 28 meters
Ⓙ 56 meters

5. Samantha opened a bank account in June and deposited some money. She deposited twice that amount in August. At the end of August, Samantha had less than $600 in her account. If she made no other withdrawals or deposits, which inequality could be used to determine the maximum amount Samantha could have deposited in June?

Ⓐ $2x < 600$
Ⓑ $2x > 600$
Ⓒ $3x < 600$
Ⓓ $3x > 600$

6. Which proportion could be used to determine the ratio of the areas of these similar rectangles?



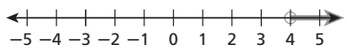
Ⓕ $\frac{2}{3}$
Ⓖ $\frac{2}{5}$
Ⓗ $\frac{4}{9}$
Ⓙ $\frac{4}{25}$

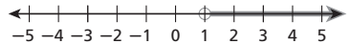
7. For which inequality is -2 a solution?

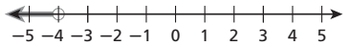
Ⓐ $2x < -4$
Ⓑ $-2x < 4$
Ⓒ $-2x > -4$
Ⓓ $-2x < -4$

8. Which graph shows the solutions of $-2(1 - x) < 3(x - 2)$?

Ⓕ

Ⓖ 

Ⓗ 

Ⓙ 

9. Which compound inequality has no solution?

Ⓐ $x > 1$ OR $x < -2$
Ⓑ $x < 1$ AND $x > -2$
Ⓒ $x < 1$ OR $x < -2$
Ⓓ $x > 1$ AND $x < -2$

Answers

1. A
2. H
3. A
4. G
5. C
6. G
7. C
8. G
9. D
10. J
11. A

To check your answer, use a different method to solve the problem from the one you originally used. If you made a mistake the first time, you are unlikely to make the same mistake when you solve the problem a different way.

10. Which inequality has the same solutions as $p < -2$?

Ⓕ $p + 1 < -2$

Ⓖ $p + 4 < 2$

Ⓗ $2p + 1 < -4$

Ⓙ $3p < -12$

11. What is the greatest integer solution of $5 - 3m > 11$?

Ⓐ 0

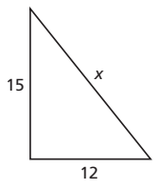
Ⓑ -1

Ⓒ -2

Ⓓ -3

Gridded Response

12. The sum of the measures of any two sides of a triangle must be greater than the measure of the third side. What is the greatest possible integer value for x?



13. After 2 years, the simple interest paid on an investment of \$2500 was \$175. What percent was the interest rate?

14. Amy's bowling score in her third game was 10 points less than her score in the first game and 5 points more than her score in the second game. The total points for all three games was no more than 275. What is the greatest number of points Amy could have scored in her first game?

15. Trevor needs a 93 on his second quiz to have a quiz average of 90. What score did Trevor receive on his first quiz?

16. The radius of a circle can be determined by the formula $r = \sqrt{\frac{A}{\pi}}$. What is the length in meters of the radius of a circle that has an area of 314 square meters? (Use 3.14 for π.)

Short Response

17. Write 2 different inequalities that have the same solution as $n > 3$ such that

a. the first inequality uses the symbol $>$ and requires addition or subtraction to solve.

b. the second inequality uses the symbol $<$ and requires multiplication or division to solve.

18. Alison has twice as many video games as Kyle. Maurice has 5 more video games than Alison. The total number of video games is less than 40.

a. Write an inequality to represent this situation.

b. Solve the inequality to determine the greatest number of video games Maurice could have. Justify each step in your solution.

19. Donna's Deli delivers lunches for \$7 per person plus a \$35 delivery fee. Larry's Lunches delivers lunches for \$11 per person.

a. Write an expression to represent the cost of x lunches from Donna's Deli. Write an expression to represent the cost of ordering x lunches from Larry's Lunches.

b. Write an inequality to determine the number of lunches for which the cost of Larry's Lunches is less than the cost of Donna's Deli.

c. Solve the inequality and explain what the answer means. Which restaurant charges less for an order of 10 lunches?

Extended Response

20. Aleya has two employment opportunities. Company A offered her a yearly salary of \$31,000. Company B offered her a similar position with a yearly salary of \$27,000 plus 2.5% commission on her total sales for the year.

a. Let x represent Aleya's total sales for the year at company B. Write an expression to represent the total income after one year at company B.

b. Use your expression from part **a** to write an inequality that could be solved to determine the amount of sales for which the yearly income at company A would be greater than that at company B.

c. Solve the inequality from part **b** and explain the meaning of the solution in relation to Aleya's decision to work for company A or company B.

d. How much more than the salary at company A would Aleya make after one year at company B if her total sales for the year were \$200,000?

Short-Response Rubric

Items 17–19

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 20

4 Points = The student correctly writes an expression in part **a,** and an inequality in part **b.** The solution in part **c** is correct, and the student correctly explains what the answer means in the context of the problem. The student gives a correct answer to part **d.**

3 Points = The student correctly writes an expression in part **a** and an inequality in part **b.** The solution in part **c** is correct, but the student does not explain the meaning of the answer in the context of the problem. The student gives a correct answer to part **d.**

2 Points = The student answers all parts correctly but does not show any work or explanation; or the student correctly answers parts **a** and **b** but does not correctly answer parts **c** and **d.**

1 Point = The student correctly writes an expression in part **a** but does not correctly answer the other parts of the problem; or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

12. J

13. A

14. G

S1a. Possible answer: $n + 5 > 8$

b. Possible answer: $-2n < -6$

S2a. $k + 2k + (2k + 5) < 40$

b. Maurice can have up to 17 videos.

S3a. $35 + 7x$; $11x$

b. $35 + 7x > 11x$

c. $x < 8.75$; Larry's costs less when 8 lunches or fewer are ordered. Donna's costs less for 10 lunches.

E1a. $27{,}000 + 0.025x$

b. $31{,}000 > 27{,}000 + 0.025x$

c. $x < 160{,}000$; Aleya should work for company A if her sales will be less than \$160,000.

d. \$1000